# Chapter 2 VECTORS

## Physical Quantity:

Physical quantity is that which can be measured by available apparatus

Observation + Measurement = Physics

Physical quantity
- 1. Scalar quantities
- 2 Vector quantities
- 3. Tensor



**(a) Scalar Quantity:** The quantity which does not change due to variation of direction is known as scalar quantity, e.g. mass, distance, time, electric current, potential, pressure etc.

## Some Important Points:

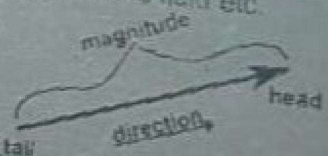
⇒ It obeys simple laws of algebra.

⇒ The scalar quantity, which is found by modulus of a vector quantity is always positive, e.g., distance, speed etc.

⇒ The scalar quantity which is found by dot product of two vectors may be negative, e.g., work, power etc.

⇒ The tensor rank of scalar quantity is zero.

**(b) Vector Quantity:** The quantity which changes due to variation of direction is known as vector quantity, e.g., displacement, velocity, electric field etc.




## Some Important Points:

⇒ Vector does not obey the laws of simple algebra.

⇒ Vector obeys the laws of vector algebra.

⇒ Vector does not obey division law. e.g $\frac{a}{b}$ is meaningless

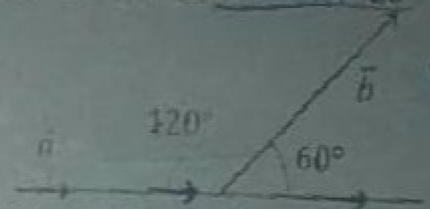
⇒ The tensor rank of vector quantity is one.

⇒ Division of a vector by a positive scalar quantity gives a new vector whose direction is same as initial vector but magnitude changes. Example $b = \frac{a}{n}$ Here n is positive

scalar. In this case direction of $\vec{a}\vec{b}$ are same to each other.

(vi) A scalar quantity never be divided by a vector quantity.

⇒ The angle between two vectors is measured by tail to tail rule. Example e.g., in the fig the angle between vector (a) and vector (b) is 60° not 120°




⇒ The angle between two vectors is always lesser or equal to 180° (i.e., $0 < \theta < 180°$)

⇒ A vector never be equal to scalar quantity

⇒ The magnitude or modulus of a vector quantity is always a scalar quantity

⇒ Two vectors are compared with respect to magnitude.

⇒ The minimum value of a vector quantity is always greater than or equal to zero

⇒ The gradient of a scalar quantity is always a vector quantity.

⇒ If a vector is displaced parallel to itself, it does not change

**(c) Tensor Quantity:** The physical quantity which is not completely specified by magnitude and direction is known as tensor quantity. Example, moment of inertia, stress etc

## Vector Algebra

⇒ $Vector + Vector = Vector$

⇒ $Vector - Vector = Vector$

⇒ $Scalar \times Vector = Vector$

⇒ $Vector \times Vector = Scalar\ or\ Vector$

⇒ $Scalar\ number \times Vector = Vector$

⇒ $Vector \div Vector = Not\ possible$

⇒ $Vector + Scalar = Vector + Scalar$

## Types of Vector

i. **Zero or empty or null vector:** The vector whose magnitude is zero and direction is indeterminate is known as zero vector. ($\vec{0}$)

### Properties of zero vector:

✓ $\vec{a} + \vec{0} = \vec{a}$

✓ $\vec{a} + \vec{b} + \vec{0} = \vec{a} + \vec{b}$

✓ $\vec{a} - \vec{a} = \vec{0}$

✓ The cross-product of two parallel vectors is always a zero vector.

✓ $n\vec{0} = \vec{0}$

✓ The velocity vector of a stationary object is zero vector

✓ The acceleration vector of object moving with uniform velocity is a zero vector

✓ The position vector of the origin of the co-ordinate axes is a zero vector.

ii. **Unit vector:** A vector of unit magnitude is known as unit vector. If $\vec{n}$ is a unit vector then $|\vec{n}| = 1$

The unit vectors along X-axis, Y-axis and Z-axis are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$

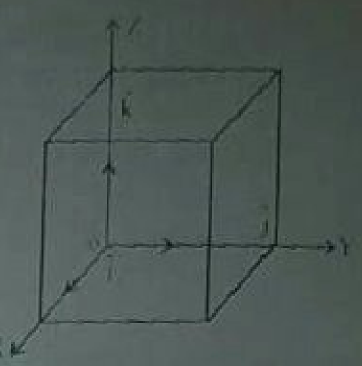
### Some important points:

✓ The angle between Vector and unit vector is 0°.

✓ The dot product of vector with its unit vector is A.

$$A = |A||\hat{A}|\cos\theta$$

$$|A|(1)\cos 0° = |A|(1) = 1 \Rightarrow \vec{A} = A$$




- ✓ Unit vector can not be expressed in any form like kg, matter or tone etc...
- ✓ $\hat{i} \neq \hat{j} \neq \hat{k}$
- ✓ $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$
- ✓ Vector = (magnitude) (direction)
- ✓ Vector = (magnitude) (unit vector)
- ✓ The unit vector along a vector $\vec{a}$ is $\hat{n} = \frac{\vec{a}}{|\vec{a}|}$ → Vector / Magnitude
- ✓ Some important unit vectors

$\hat{i} \rightarrow x - \text{axis}$

$\hat{j} \rightarrow y - \text{axis}$

$\hat{k} \rightarrow z - \text{axis}$

$\hat{n} \rightarrow$ normal to surface area

$\hat{r} \rightarrow$ unit vector of position vector

- ✓ Unit vector has no unit
- ✓ Sometime in MCQ you are asked; Which of the following represent unit vector?... and you have given different vector.

The simple trick is first solving or calculate the magnitude of all, as we are familiar about that unit vector has magnitude equal to 1, so the vector whose magnitude comes out to be 1 will be unit vector. (arsal)

**Parallel vector:**

**Like parallel vector:** Same direction irrespective of magnitude and starting position




**Unlike parallel vector:** Having opposite direction irrespective of magnitude and Starting position. $\theta = 180^\circ$




## Some important points:

- ✓ If vectors are parallel, then their unit vectors are same to each other. Suppose $\vec{a}$ and $\vec{b}$ are parallel vectors, then $\frac{\vec{a}}{|\vec{a}|} = \frac{\vec{b}}{|\vec{b}|}$.
- ✓ The angle between like parallel vectors is zero.
- ✓ The angle between unlike parallel vectors is 180°.
- ✓ The magnitude of parallel vectors may or may not be same.
- ✓ If the magnitude of like parallel vectors is same, then the vectors are known as equal vectors.

**Polar vectors:** The vectors related to translatory motion of a body are known as polar vectors, e.g., linear velocity, linear momentum etc.

**Axial or pseudo vectors:** The vectors which are rotatory to rotatory motion of a body are known as axial vectors, e.g., torque, angular velocity etc.

**Negative vector:** Same magnitude opposite in direction irrespective of starting point.




$\vec{A}$ and $\vec{B}$ are negative vector $\vec{A} = -\vec{B}$

**Thus, negative vectors is a special case of unlike vectors.**

1) - East = west
2) - West = East

3) -North = South
4) -South = North
5) $\vec{x} = -\vec{y}$ → Here y is positive but X is the negative vector of y
6) $\vec{y} = -\vec{x}$ → Here −x is positive but y is negative of X

Note: The vector which we take along a reference axis must be positive

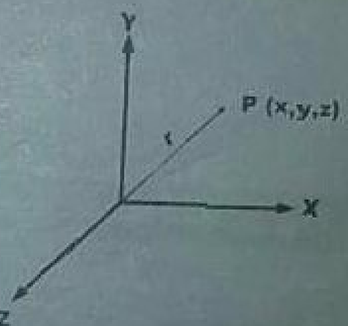
Reference axis = Positive vector

7) **Equal vector:** Having same direction and magnitude irrespective of starting point. $\vec{A} = \vec{B}$ and $-\vec{A} = -\vec{B}$

Suppose $\vec{a}$ and $\vec{b}$ are equal vectors the $|\vec{a}| = |\vec{b}|$

8) **Position vector:** The position vector of an object is the vector from the origin O of the co-ordinate system to the position of the object

⇒ The position vector gives straight line distance of the object from the origin O (starting point)
⇒ It gives the direction of the object w.r.t origin
⇒ Symbolic representation: $\vec{r}$
⇒ Uses of position vector is, it is use to locate a point in space/plane, it describes position of moving particles or body, it is used in electricity and magnetism to specify electric or magnetic field at a certain point.
⇒ Graphical representation:




Position vector is represented by a straight line in such a way that its tail coincides with origin o and its head joins with the position of a point 'p'

Position vector in 2 dimensional plane:

$\vec{r} = x\hat{i} + y\hat{j}$

Position vector in 3 dimensional plane:

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

For magnitude:

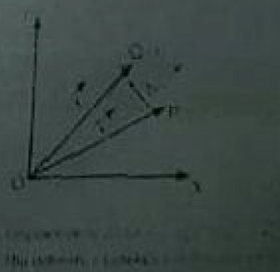
$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

For direction:

$\tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\frac{y}{x}$

9) **Displacement vector:**

Displacement is a vector quantity and so-called displacement vector So, displacement vector is a vector that points from object's initial position and whose magnitude is equal to straight line distance between the two points.



⇒ It is not drawn from origin of co-ordinate system
⇒ This vector gives us displacement between two points in a space
⇒ It is also called free vector
⇒ Symbolical representation:

$\underline{\Delta\vec{r} \text{ or } \vec{d}}$

⇒ Formula

$$\Delta\vec{r} = r_2 - r_1 \rightarrow *$$

or

$$\Delta\vec{r} = \vec{r_1} - \vec{r_2} \rightarrow *$$

In above both equations, the magnitude is same but the direction is not same. So

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1} \Rightarrow \vec{r_2} = \vec{r_1} + \Delta r$$
$$\Delta\vec{r} = \vec{r_1} - \vec{r_2} \Rightarrow \vec{r_1} = \vec{r_2} + \Delta r$$

⇒ For locating the point, the concerned vector must be taken positive.

⇒ For magnitude

$$|\Delta\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

## PRACTICE:

$$r_1 = x_1\hat{i} + y_1\hat{j}$$
$$r_2 = x_2\hat{i} + y_2\hat{j}$$
$$\Delta r = r_2 - r_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$
$$\Delta r = r_1 - r_2 = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j}$$

## EXAMPLE:

$$p_1 = (1,2) \text{ and } p = (3,4)$$
$$\Delta_r = \vec{r_1} - \vec{r_2} = (1-3)\hat{i} + (2-4)\hat{j}$$
$$= -2\hat{i} + (-2\hat{j})$$
$$= -2\hat{i} - 2\hat{j} \rightarrow \text{negative sign show position of direction}$$
$$\Rightarrow \Delta\vec{r} = \vec{r_2} - \vec{r_1} = 2\hat{i} + 2\hat{j}$$

## Is current a scalar or a vector quantity?

Electric current is a SCALAR quantity! Sure, it has magnitude and direction, but it still is a scalar quantity!

Confusing? Let us see why it is not a vector.

First let us define a vector! A physical quantity having both magnitude and a specific direction is a vector quantity. Is that all? No! This definition is incomplete! A vector quantity also follows **the triangle law of vector addition.**

Let us understand that with a simple example! Say you are at home right now! From there you go to school and then you go shopping to some supermarket (ambrikalla). So now you have moved from points A to B to C! Now when you come back home again, what is your net displacement? Its zero, because in the real sense of the word displacement, you went nowhere! You are still at your initial position! So now, net result along the path A-B-C-A is zero! This is the triangle law of vector addition!

Now consider a triangular loop in an electric circuit with vertices A, B and C. The current flows from A→ B, B→C and C→A. Now had current been a vector quantity, following the triangle law of vector addition, the net current in the loop should have been zero! But that is not the case, right? You won't be having a very pleasant experience if you touch an exposed high current loop. So that is why, electric current is a scalar quantity!

## Important Information related to Trigonometry

$$\sin\theta = \frac{\text{opposite}}{\text{Hypotenuse}} = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r} = \frac{x}{1} = x$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

| angle | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ | $2\pi/3$ | $3\pi/4$ | $5\pi/6$ | $\pi$ |
| sin | $\frac{\sqrt{0}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{4}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{0}}{2}$ |
| cos | $\frac{\sqrt{4}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{0}}{2}$ | $-\frac{\sqrt{1}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{4}}{2}$ |
| tan | $\frac{\sqrt{0}}{\sqrt{4}}$ | $\frac{\sqrt{1}}{\sqrt{3}}$ | $\frac{\sqrt{2}}{\sqrt{2}}$ | $\frac{\sqrt{3}}{\sqrt{1}}$ | – | $-\frac{\sqrt{3}}{\sqrt{1}}$ | $-\frac{\sqrt{2}}{\sqrt{2}}$ | $-\frac{\sqrt{1}}{\sqrt{3}}$ | $-\frac{\sqrt{0}}{\sqrt{4}}$ |
| cot | – | $\frac{\sqrt{3}}{\sqrt{1}}$ | $\frac{\sqrt{2}}{\sqrt{2}}$ | $\frac{\sqrt{1}}{\sqrt{3}}$ | 0 | $-\frac{\sqrt{1}}{\sqrt{3}}$ | $-\frac{\sqrt{2}}{\sqrt{2}}$ | $-\frac{\sqrt{3}}{\sqrt{1}}$ | – |
| csc | – | $\frac{2}{\sqrt{1}}$ | $\frac{2}{\sqrt{2}}$ | $\frac{2}{\sqrt{3}}$ | $\frac{2}{\sqrt{4}}$ | $\frac{2}{\sqrt{3}}$ | $\frac{2}{\sqrt{2}}$ | $\frac{2}{\sqrt{1}}$ | – |
| sec | $\frac{2}{\sqrt{4}}$ | $\frac{2}{\sqrt{3}}$ | $\frac{2}{\sqrt{2}}$ | $\frac{2}{\sqrt{1}}$ | – | $-\frac{2}{\sqrt{1}}$ | $-\frac{2}{\sqrt{2}}$ | $-\frac{2}{\sqrt{3}}$ | $-\frac{2}{\sqrt{4}}$ |

## Multiplication of a vector by a number:

i. If we multiply a vector $\vec{A}$ by a real positive number 'n' we get the vector $n\vec{A}$ which has the same direction as $\vec{A}$ and magnitude nA.

$$\text{i.e ... } n\vec{A} = n(\vec{A})$$

And so, the magnitude of vector becomes n times while its direction remains unchanged.

ii. If vector is multiplying by a negative number (i.e... –n) the magnitude of vector becomes nA but direction is opposite to that of $\vec{A}$

$$\text{i.e ... } -n\vec{A} = -n(\vec{A})$$

### MCQ:

⇒ If we multiply vector $\vec{A}$ by 3 we get another vector $\vec{B}$ which has same direction as that of A, but has 6 times the magnitude. $\vec{B} = 3\vec{A}$

⇒ If we multiply vector $\vec{A}$ by 2 we get another vector $\vec{B}$ which is parallel to A, but in opposite direction and magnitude of 2A. $\vec{B} = -2\vec{A}$

## Addition of vector

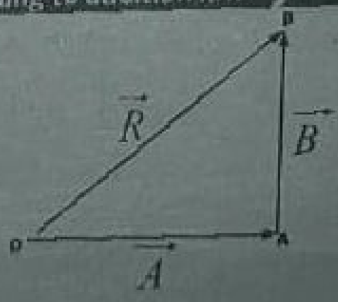
If we add two or more than two vectors and get a single vector whose effect is same as that of individual vectors that acts.

→ There are two methods for addition of vector

⇒ Head to tail rule of vector addition (graphical method), this method is less accurate but gives us more information i.e.; point to point information.

⇒ Addition of vectors by rectangular component method (analytical method). it is more accurate but gives us more information.

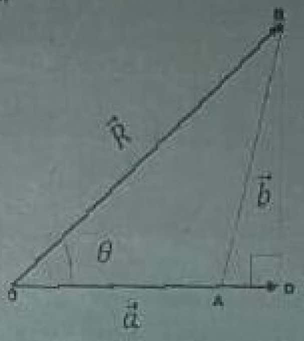
### (a) According to addition law



$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} \Rightarrow \text{i.e. } \vec{a} + \vec{b} = \vec{R}$$

## (b) Magnitude of $\vec{R}$:

Let $\alpha$= angle between $\vec{a}$ and $\vec{b}$ then,




$$|\vec{R}| = \sqrt{a^2 + b^2 + 2ab\cos\alpha}$$

## (c) Direction of $\vec{R}$:

Let resultant $\vec{R}$ makes an angle $\theta$ with $\vec{a}$. Then,

$$\tan\theta = \frac{b\sin\alpha}{a + b\cos\alpha}$$

- $|\vec{R}|_{max} = a + b$
- $|\vec{R}|_{min} = a - b$
- $a + b > R > a - b$
- Vector addition obeys commutative law, i.e. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
- Vector addition obeys associative law, i.e. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
- we can move the vector from one place to another but we cannot change magnitude and direction of a vector while moving it.
- Resultant vector of two or more vector is a single vector which produces the same effect as produced by individual vector together (will be explained further in end of this chapter).
- The advantage of resultant vector is that we replace many vectors by a single vector.
- There are triangle law of vector addition and second is parallelogram law of vector addition.

## Lets explain some special cases of addition of vector:

i) ***When the vector acts along the same direction (when angle is 0°)***

$$R = \sqrt{a^2 + b^2 + 2ab\cos 0^0} = \sqrt{(a+b)^2}$$

$$\tan\theta = \frac{b\sin 0^0}{a + b\cos 0^0} = \frac{b(0)}{a + b(1)} \therefore \alpha = 0^0$$

Therefore, the magnitude of the resultant of two vectors in the same direction is equal to the sum of magnitude of two vector and the direction of resultant is along the direction of $\vec{a}$ and $\vec{b}$.

ii) ***When the vector is at right angles (when angle is 90o)***

$$R = \sqrt{a^2 + b^2 + 2ab\cos 90^0} = \sqrt{(a+b)^2}$$

$$\tan\theta = \frac{b\sin 90^0}{a + b\cos 90^0} = \frac{b(1)}{a + b(0)} = \frac{Q}{p} \ldots \tan\alpha = \frac{b}{a}$$

iii) ***When the vector acts along opposite direction (when angle is 180o)***

$$R = \sqrt{a^2 + b^2 + 2ab\cos 180^0} = \sqrt{(a-b)^2} \text{ so } R = a - b$$

$$\tan\theta = \frac{b\sin 180^0}{a + b\cos 180^0} = \frac{b(0)}{a + b(-1)} = 0$$

$$\alpha = 0^0 \text{ or } 180^0$$

Therefore, the magnitude of the resultant of two vector acting in the opposite direction is equal to the difference in the magnitude of the vectors. The resultant acts in the direction of largest vector.

## Some important points from vector addition:

⇒ *Addition of vectors are commutative;*

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

⇒ $\vec{R} = \vec{A} + \vec{B}$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$\theta \Rightarrow$ angle between two vectors,

$$0^0 \le \theta \le 180^0$$

Note that in vector operation i.e. addition, subtraction or multiplication we will always take inner angle or smaller angle.

⇒ $|\vec{R}| \le A + B$

case (i): $|\vec{R}| = A + B$

if $\theta = 0^0$ (maximum possible sum)

case (ii): $|\vec{R}| < A + B$

for angle other that $0^0$

case (iii): $|\vec{R}| = A - B$ or $B - A$

if $\theta = 180^0$ (minimum possible result)

⇒ *We know that;*

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

if $\theta = 90^0$

$$R = \sqrt{A^2 + B^2}$$

⇒ *From the above discussion we can generalize the resultant;*

$$\text{Minimum} \le R \le \text{Maximum}$$

**MCQ:** if $\vec{A}$ = 7 units, $\vec{B}$ = 10 units

*Then maximum possible sum = 10 + 7 = 17 units*

*And minimum possible sum = 10 - 7 = 3 units*

**MCQ:** x = 5 units, y = 12 units

Then possible sum x + y = ____________

a) 14 units
b) 4 units
c) 18 units
d) 8 units

**EXPLANATION:**

$$\text{Min} \le R \le \text{Max}$$

$$7 \le R \le 17$$

That is 14 units

**MCQ:**

The angle between two vectors of equal magnitude is 120°. then magnitude of their resultant is equal to either of them. R = F   θ = 120°

## Subtraction of vectors

⇒ Let two vectors $\vec{a}$ and $\vec{b}$ make an angle θ with each other. We define

$$\vec{R} = \vec{a} - \vec{b}$$

⇒ Then, $\vec{R}$ is resultant of $\vec{a}$ and $(-\vec{b})$.

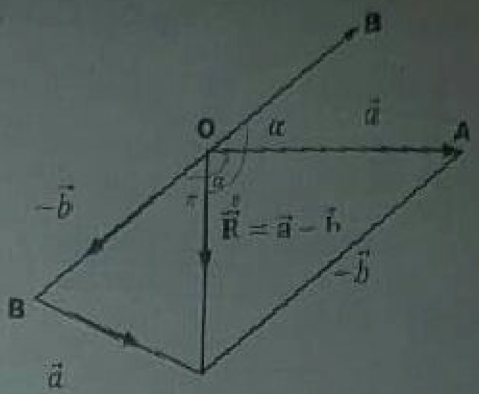
⇒ The angle between $\vec{a}$ and $-\vec{b}$ is $\pi - \alpha$

⇒ According to parallelogram law of vectors, the magnitude of $\vec{R}$

$= |\vec{R}| = \sqrt{a^2 + b^2 + 2ab\cos(\pi - \alpha)}$

$= |\vec{R}| = \sqrt{a^2 + b^2 - 2ab\cos\alpha}$

And $\tan\alpha = \frac{b\sin(\pi-\alpha)}{a+b\cos(\pi-\alpha)} = \frac{b\sin\alpha}{a-b\cos\alpha}$



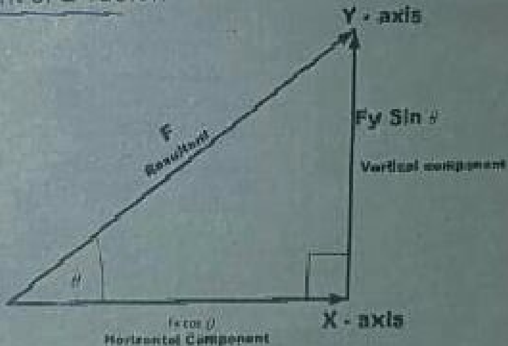

## RESOLUTION OF VECTOR INTO ITS RECTANGULAR COMPONENTS:

⇒ The process of splitting single vector into two or more than two components is called resolution of vector.

**Component:** the effective value of a vector in a particular direction is called component of vector in that direction.

⇒ It is projection of vector on any axis of co-ordinate system.

⇒ Importance of resolution of vector is, due to resolution of vector into its component we know about the effective component of a vector.




$F = vector$

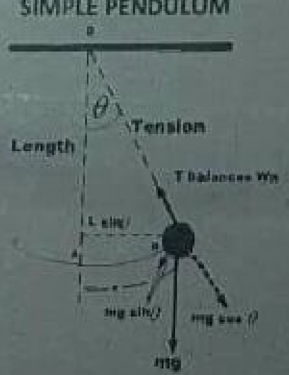
$F_x = Component\ of\ F\ along\ x - axis$

$F_y = Component\ of\ F\ along\ y - axis$

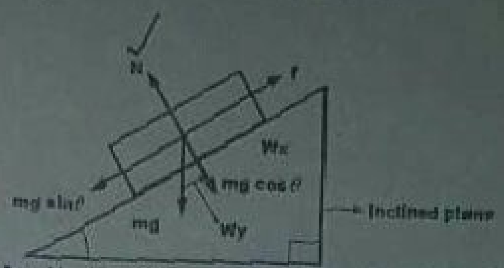
⇒ ***Effective components of simple pendulum;***

$w_y = mg\sin\theta;$

*effective component so show effects on* $w_y$




⇒ ***Effective component on inclined plane;***




*in this case $W_x$ is balanced by N.. but $W_y$ is not balanced so it an efffective component*

⇒ Same as that if we have a block moving on straight path then their effective component will be $F_x$ and vertical component $F_y$ and W balance each other. And during motion W act downwards.

⇒ Remember that the horizontal component cancels out the horizontal one but not vertically i.e.. x- component will cancel only x-component, or, x-axis will cancel out x-axis).

⇒ Vertical component will cancel out only vertical component but not the horizontal one.

*If F and θ is known for components we have trigonometric ratios;*

(i) **For x – component:**

$$\cos\theta = \frac{F_x}{F_y} \Rightarrow F\cos\theta = F_x \Rightarrow F_x = F\cos\theta$$

$$\textbf{vectorial form} :: \vec{F_x} = F\cos\theta\,\hat{\imath}$$

(ii) **For y – component:**

$$\sin\theta = \frac{F_y}{F} \Rightarrow F\sin\theta = F_y \Rightarrow F_y = F\sin\theta$$

$$\textbf{vectorial form} :: \vec{F_y} = F\cos\theta\,\hat{\jmath}$$

(iii) **For vector "F":**

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$$

Note that: $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$

Here $F_x$ *and* $F_y$ represent magnitude while $\hat{\imath}$ *and* $\hat{\jmath}$ are the unit vector.

Magnitude is scalar, so we cannot represent it as; $\vec{F} = \vec{F_x}\hat{\imath} + \vec{F_y}\hat{\jmath} \rightarrow$ *so it is incorrect*

⇒ **If $\vec{F}$ is unknown:**

If x and y components are known;

So;

$$F = \sqrt{F_x^2 + Fy^2} \rightarrow \text{in 2D}$$

$$F = \sqrt{F_x^2 + Fy^2 + Fz^2} \rightarrow \text{in 3D}$$

⇒ **For direction of $\vec{F}$:**

$$\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1}\frac{F_y}{F_x}$$

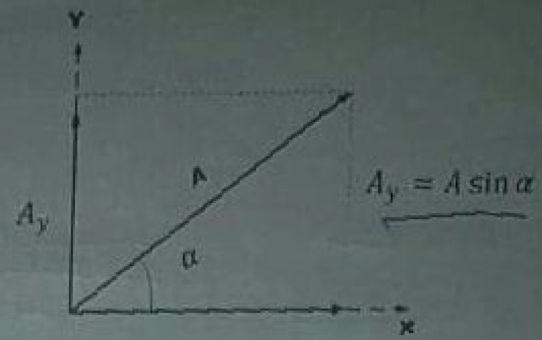
## SOME IMPORTANT MCQ POINTS:

$$\hat{F} = \cos\theta\hat{\imath} + \sin\theta\hat{\jmath} \rightarrow \text{IF } \theta \rightarrow \text{witH Horizental axis}$$

$$\hat{F} = \sin\theta\hat{\imath} + \cos\theta\hat{\jmath} \rightarrow \text{if } \theta \rightarrow \text{witH vertical axis}$$

## Different cases:

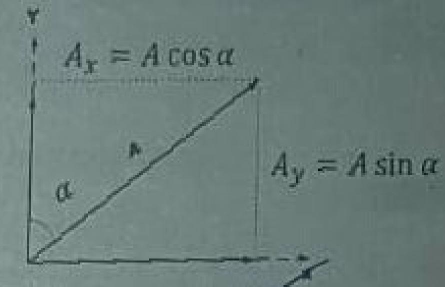
(i) **Case (i):**




In this case $A_x = A\cos\theta \rightarrow F_x = F\cos\theta$

$A_y = A\sin\theta \rightarrow F_y = F\sin\theta$

(ii) **Case (ii):**




In this case $F_x = F\sin\theta$ while $F_y = F\cos\alpha$

## MULTIPLE CHOICE QUESTIONS

1) A vector F making an angle of 30 degree with y-axis then its vertical compoenet is?
- a) F sin 30°
- b) F sin 60°
- c) F cos 30°
- d) F cos 60°

**EXPLANATION:** option c is correct, because if we draw 3o degree from y-axis (case 2nd) then its vertical component $Fy = \cos\theta$

2) If $\vec{F} = 10N\hat{i}$ then $F_x$_______? And $F_y$_______?
- a) 10 N and 0N
- b) 5N and 5N
- c) 3N and 3N
- d) 0N and 10N

**EXPLANATION:** option a is correct, we have studied that î unit vector acts along x-axis, so $F_x$= 10N, also we know that $F_x$ and $F_y$ these components are independent of each other, so $F_y$ =0N

3) If force of 20N acts along the x-axis then the force acts on y-axis will be________?

(a) 20N (b) 30N (c) 0N (d) 10N

**EXPLANATION:** 0 N is correct because magnitude of x-axis iis given which is 20N but there is no magnitude of y-axis given, and they are independent of each other so we cannot tell anything about magnitude of y-axis so simply it is zero.

4) If $\vec{F}$ = 3N while $F_x$ = 2N and $F_y$ = 1N, So, $F_z$ will be______________?

a) 6N b) 3N c) 1N d) 2N (correct)

**EXPLANATION:**

**1st method:**

$$F = \sqrt{Fx^2 + Fy^2 + Fz^2} \Rightarrow 3 = \sqrt{2^2 + 1^2 + Fz^2} \Rightarrow 3 = \sqrt{4 + 1 + Fz^2}$$

$$3 = \sqrt{4+1+4} \rightarrow \text{and } \sqrt{4} = 2 \text{(so 2 is correct)}$$
$$3 = \sqrt{3+4} \Rightarrow 3 = \sqrt{9} \Rightarrow 3 = 3$$
So, **2** is correct

***2nd method:***

$$|F| = \sqrt{Fx^2 + Fy^2 + Fz^2} \Rightarrow 3 = \sqrt{2^2 + 1^2 + Fz^2} \Rightarrow 3 = \sqrt{5 + Fz^2}$$
squaring both the sides
$$(3)^2 = \left(\sqrt{5 + F_z^2}\right)^2 \Rightarrow 9 - 5 = Fz^2 \Rightarrow 4 = F_z^2 \Rightarrow 2 = F_x$$

If F = 10N ($F_x$ = 6N and $F_y$ = 8N) then $F_z$=__________?
a) 1N
b) 2N
c) 3N
d) 0N (correct)

**EXPLANATION:**

**Simple trick;**

$$|10| = \sqrt{30 + 64 + ?} \Rightarrow \sqrt{100 + ?}$$
so the unknown component must be 0N

## ADDITION OF VECTOR BY RECTANGULAR COMPONENT METHOD (ANALYTICAL METHOD):

⇒ In this method we will add corresponding components;
i.e.
⇒ x component with x component
⇒ y component with y component
⇒ z component with z component
to get the corresponding component is known as rectangular component.

**EXAMPLE:**

$$A = 3\hat{i} + 2\hat{j} \text{ and } B = \hat{i} + 5\hat{j}$$
$$R = \vec{A} + \vec{B} \Rightarrow R = (3+1)\hat{i} + (2+5)\hat{j} \Rightarrow R = 4\hat{i} + 7\hat{j}$$

⇒ it's means that 1st we resolve the vector into its component.
⇒ Then we will add corresponding components.

⇒ *In plane (2D)*
X-component with x-component
Y-component with y-component

⇒ *In space (3D)*
x- Component with x-component
Y-component with y-component
Z-component with z-component

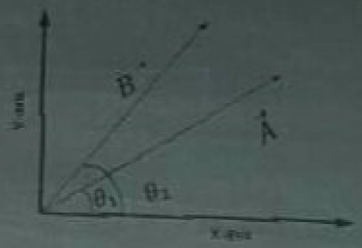
## METHOD:

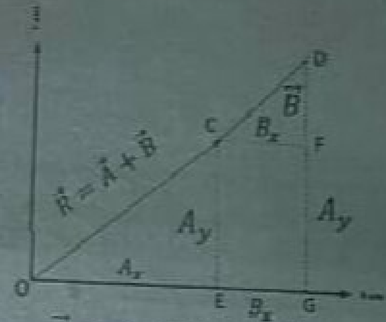
Suppose we have two vectors;
$$\vec{A} \text{ and } \vec{B}$$
Where;
$$\vec{A} = A_x\hat{i} + A_y\hat{j} \Rightarrow \vec{B} = B_x\hat{i} + B_y\hat{j}$$
So;
$$\vec{R} = \vec{A} + \vec{B}$$
Or;
$$\vec{R} = R_x + R_y$$
$$R = \vec{A} + \vec{B}$$
$$\vec{R} = (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j})$$
$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$
$$\vec{R} = (R_x)\hat{i} + (R_y)\hat{j}$$

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

Or;

$$\vec{R} = \vec{R_x} + \vec{R_y} \rightarrow \text{for 2D (plane)}$$
$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k} \rightarrow \text{for 3D (space)}$$





$$\vec{A} \rightarrow A_x(OE), A_y(FG)$$
$$\vec{B} = B_x(EG), B_y(DF)$$
$$R \rightarrow R_x + R_y$$

**For Horizental Rx:**

From the figure;

$$OG = OE + EG \Rightarrow \vec{R_x} = \vec{A_x} + \vec{B_x} \rightarrow *$$

**For vertical Ry:**

From the figure;

$$GD = FG + DF \Rightarrow \vec{R_y} = \vec{A_y} + \vec{B_y} \rightarrow *$$

**Magnitude of R:**

As;

$$\vec{R} = \vec{R_x} + \vec{R_y} \Rightarrow \vec{R} = (\vec{A_x} + \vec{B_x})^2 + (\vec{A_y} + \vec{B_y})$$

**By pythagoras theorem:**

$$(R)^2 = (\vec{A}_x + \vec{B_x})^2 + (\vec{A_y} + \vec{B_y})^2 \Rightarrow \sqrt{(\vec{A}_x + \vec{B_x})^2 + (A_y + B_y)^2}$$

**For direction:**

$$\tan\theta = \frac{R_y}{R_x} \Rightarrow \tan\theta = \frac{A_y + B_y}{A_x + B_x} \Rightarrow \theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

## MULTIPLE CHOICE QUESTIONS:

1) $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$, where $\vec{D} = 3\hat{i} - \hat{j} - \hat{k}$

Then what will be $\vec{R}$____? and what will be $|\vec{R}|$______?

EXPLANATION:

$$\vec{R} = u\hat{i} + \hat{j} + 2\hat{k}$$

AND:

$$|R| = \sqrt{4^2 + 1^2 + 2^2} = \sqrt{21} \text{ So;} \rightarrow |\vec{R}| = \sqrt{21}$$

2) $\vec{A} = 2\hat{i} - \hat{j}$ wHere $\vec{B} = -\hat{i} + \hat{j}$

Then what will be $\vec{R}$_____? And find $|\vec{R}|$_______?

EXPLANATION:

$$\vec{R} = (A_x + B_x) + (A_y + B_y)$$

$$\vec{R} = \hat{i}$$

$$\text{and; } |\vec{R}| = \sqrt{R^2}$$

$$\text{so; } |\vec{R}| = 1$$

3) If $\vec{A} + \vec{B} = \hat{i} + \hat{j}$, and $\vec{A} - \vec{B} = 2\hat{i} - 4\hat{j}$ then what will be $\vec{A} =$ ____? And $\vec{B} =$ ____?

EXPLANATION:

**TRICK:**

$1^{st}$ add;

$$A + B = \hat{i} + \hat{j}$$
$$A + B = 2\hat{i} - 4\hat{j}$$
$$2A \quad = 3\hat{i} - 3\hat{j}$$

$$\vec{A} = \frac{3\hat{i}}{2} - \frac{3\hat{j}}{2}$$

⇒ THEN subtract;

$$A + B = \hat{i} + \hat{j}$$
$$+A + B \mp 2\hat{i} - 4\hat{j}$$
$$2B \quad = -\hat{i} - 5\hat{j}$$

$$\vec{B} = -\frac{1}{2}\hat{i} + \frac{5}{2}\hat{j}$$

**FOR DIRECTION OF 'R':**

$$\theta = \tan^{-1}\left|\frac{\vec{A_y} + \vec{B_y}}{\vec{A_x} + \vec{B_x}}\right| \Rightarrow \theta = \tan^{-1}\left|\frac{\vec{R_y}}{\vec{R_x}}\right|$$

⇒ ***But the big θ is that what should be the angle with respect to quadrents.***

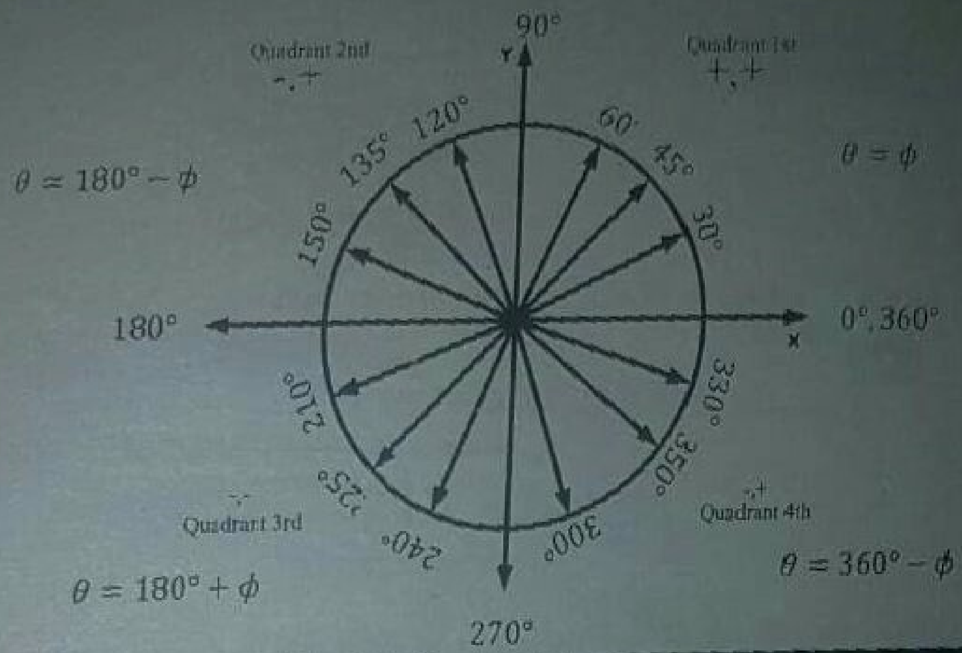
⇒ ***$1^{st}$ (i) $2^{nd}$ (ii) $3^{rd}$ (iii) and $4^{th}$ (iv) QUADRENTS angle behavior***

## Two things to be considered during studying particular angle in a respective QUADRENTS.

i. θ should always take from anticlockwise
ii. And θ should be always taken from +ve-axis.

| Quadrant II "Science" | Quadrant I "All" |
|---|---|
| sin, cosec + <br> cos, sec − <br> tan, cot − | sin, cosec + <br> cos, sec + <br> tan, cot + |
| **Quadrant III "Teachers"** | **Quadrant IV "Crazy"** |
| sin, cosec − <br> cos, sec − <br> tan, cot + | sin, cosec − <br> cos, sec + <br> tan, cot − |

(y-axis vertical, x-axis horizontal)




## 1ST Quadrant:

$$\varphi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\text{so } \varphi = \theta$$

**EXAMPLE:**

$$R = \hat{i} + \hat{j} \quad \text{THEN } \theta = ____?$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) \Rightarrow \theta = \tan^{-1}(1) \rightarrow \theta = 45^0$$

$$\text{so } \varphi = \theta \text{ at } 45^0$$

## 2ND Quadrant:

$$\varphi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

So;

$$\theta = -\varphi + 180^0 \Rightarrow \text{so } \theta = 180^0 - \varphi$$

**EXAMPLE:**

$$R = -\hat{i} + \hat{j} \text{ THEN } \theta = ____?$$

$$\theta = \tan^{-1}\left(\frac{-1}{1}\right) \Rightarrow \theta = \tan^{-1}(-1) \rightarrow \theta = 45^0$$

But;

$$\theta = 180^0 - \varphi \rightarrow \theta = 180^0 - 45^0 \Rightarrow \theta = 135^0$$

## 3RD Quadrant:

$$\varphi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\theta = \varphi + 180^0$$

**EXAMPLE:**

$$R = -\hat{i} - \hat{j} \text{ then } \theta = ____?$$

$$\theta = \tan^{-1}\left(\frac{-1}{-1}\right) \Rightarrow \theta = \tan^{-1}(1) \rightarrow \theta = 45^0$$

$$\theta = 180^0 + \varphi$$

So

$$\theta = 225^0$$

## 4TH Quadrant:

$$\varphi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$
$$\Rightarrow \theta = 360^\circ - \varphi$$

**EXAMPLE:**

$$R = \hat{i} - \hat{j} \text{ then } \theta = ______?$$
$$\theta = \tan^{-1}\left(\frac{-1}{1}\right) \Rightarrow \theta = \tan^{-1}(-1) \rightarrow \theta = 45^0$$

But;

$$\theta = 360^\circ - \varphi$$

So;

$$\theta = 315^0$$

## How to find that the angle lies in which "QUADRANT" easily:

### Case (i):

⇒ if $R_x = +ve$ and $R_y = +ve$, this means that if x-axis and y-axis both are +ve then θ will lie in 1st quadrant

$$\theta = \varphi \Rightarrow R_x \text{and } R_y = +ve$$

MCQ: if $R = \hat{i} + \hat{y}$ then θ will lies in 1st quadrant
MCQ: if $R = 2\hat{i} + 3\hat{y}$ then θ will lies in 1st quadrant
⇒ Because in 1st quadrant x and y axis both are +ve.

### Case (ii):

⇒ if $R_x = -ve$ and $R_y = +ve$, this means that x is negative while y is positive then θ will lie in 2nd quadrant

$$\theta = 180^0 - \varphi \Rightarrow R_x = -ve \text{ while } R_y = +ve$$

MCQ: if $R = -\hat{i} + \hat{y}$ then θ will lies in 2nd quadrant
MCQ: if $R = -2\hat{i} + 3\hat{y}$ then θ will lies in 2nd quadrant
⇒ Because in 2nd quadrant x axis is negative while y is positive.

### Case (iii):

⇒ if $R_x = -ve$ and $R_y = -ve$, this mean that x and y both are negative, then in this case θ will lie in 3rd quadrant.

$$\theta = 180^0 + \varphi \Rightarrow R_x \text{ and } R_y \text{ both are negative } (-ve)$$

MCQ: if $R = -\hat{i} - \hat{y}$ then θ will lies in 3rd quadrant
MCQ: if $R = -2\hat{i} - 3\hat{y}$ then θ will lies in 3rd quadrant
⇒ Because in 3rd quadrant x and y axis both are -ve.

### Case(iv):

⇒ if $R_x = +ve$ and $R_y = -ve$, this mean that x is positive while y-axis is negative in this case θ will lies in 4th quadrant.

$$\theta = 360^\circ - \varphi \Rightarrow R_x = +ve \text{ while } R_y = -ve$$

MCQ: if $R = \hat{i} - \hat{y}$ then θ will lies in 3rd quadrant.
MCQ: if $R = 2\hat{i} - 3\hat{y}$ then θ will lies in 3rd quadrant.
⇒ Because in 4th quadrant x component is +ve while y axis is negative.

**P R O B L E M - S O L V I N G H I N T S ; Adding Vectors**

When you need to add two or more vectors, use this step-by-step procedure:
• Select a coordinate system that is convenient. (Try to reduce the number of components you need to calculate by choosing axes that line up with as many vectors as possible.)
• Draw a labeled sketch of the vectors described in the problem.
• Find the *x* and *y* components of all vectors and the resultant components (the algebraic sum of the components) in the *x* and *y* directions.
• If necessary, use the Pythagorean theorem to find the magnitude of the resultant vector and select a suitable trigonometric function to find the angle that the resultant vector makes with the *x* axis.

Tangents on Calculators Generally, the inverse tangent function on calculators provides an angle between - 90° and + 90°. As a consequence, if the vector you are studying lies in the second or third quadrant, the angle measured from the positive x axis will be the angle your calculator returns plus 180°.

**Example: The Sum of Two Vectors**

Find the sum of two vectors **A** and **B** lying in the $xy$ plane and given by $A = (2.0\hat{i} + 2.0\hat{j})m$ and $B = (2.0\hat{i} + 4.0\hat{j})m$?

**Solution** You may wish to draw the vectors to conceptualize the situation. We categorize this as a simple plug-in problem.

Comparing this expression for **A** with the general expression $A = A_x\hat{i} + A_y\hat{j}$, we see that $A_x = 2.0$ m and $A_y = 2.0$ m.

Likewise, $B_x = 2.0$ m and $B_y = -4.0$ m. We obtain the resultant vector **R**, using Equation:

$$R = A + B \Rightarrow (2.0\hat{i} + 2.0\hat{j})m + (2.0\hat{i} + 4.0\hat{j})m = (4.0\hat{i} - 2.0\hat{j})m$$

or

$$R_x = 4.0m \quad and \quad R_y = -2.0m$$

The magnitude of **R** is found using Equation;

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0m)^2 + (-2.0m)^2} = \sqrt{20}m = 4.5m$$

We can find the direction of **R** from Equation;

$$tan\theta = \frac{R_y}{R_x} = -\frac{2.0m}{4.0m} \Rightarrow -0.50m$$

Your calculator likely gives the answer - 27° for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° clockwise from the x axis. Our standard form has been to quote the angles measured counterclockwise from the + x axis, and that angle for this vector is $\theta = 333°$

## Scalar product or dot product

**PROPERTIES:**

⇒ The scalar or dot product depends upon; $\vec{A}\cdot\vec{B} = AB\cos\theta$

⇒ If $0 < \theta < 90° \Rightarrow \vec{A}\cdot\vec{B} = +ve \Rightarrow$ POSITIVE

⇒ If $\theta = 0° \Rightarrow$ then $\vec{A}\cdot\vec{B} = AB \Rightarrow$ Maximum

⇒ If $\theta = 90° \Rightarrow$ then $\vec{A}\cdot\vec{B} = 0 \Rightarrow$ ZERO

⇒ If $90° < \theta < 180° \Rightarrow \vec{A}\cdot\vec{B} = -AB \Rightarrow$ NEGATIVE

⇒ If $\theta = 180° \Rightarrow$ then $\vec{A}\cdot\vec{B} = -AB \Rightarrow$ minimum resultant (in this case the magnitude is not negative but we say that magnitude as a whole cannot be negative, this negative only tell us the opposing property like that in work done against friction force)

Between 90° → 180° all values of $\cos\theta = NEGATIVE$

⇒ Dot product of two vectors is commutative; $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

⇒ $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

⇒ Dot product of a vector with itself; $\vec{a}\cdot\vec{a} = a^2, \vec{B}\cdot\vec{B} = B^2$ for example: $k\cdot e = \frac{1}{2}m(\vec{v}\cdot\vec{v}) \Rightarrow k\cdot e = \frac{1}{2}mv^2$

⇒ Dot product of a two-unit vectors; $\hat{i}\cdot\hat{i} = 1,\ \hat{j}\cdot\hat{j} = 1$ and $\hat{k}\cdot\hat{k} = 1$

⇒ Orthogonal vectors; $\hat{i}\cdot\hat{j} = 0$ , $\hat{j}\cdot\hat{k} = 0$ and $\hat{i}\cdot\hat{k} = 0$

⇒ Dot product in component from: If $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ then $\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z$

⇒ The angle between vector $\vec{a}$ and $\vec{b}$ is given by $\theta = \cos^{-1}\left|\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right|$

⇒ The component of $\vec{a}$ parallel to $\vec{b}$ in vector form is given by $\vec{c} = \frac{(\vec{a}\cdot\vec{b})\vec{b}}{|\vec{a}||\vec{b}|}$

⇒ The component of $\vec{a}$ perpendicular $\vec{b}$ in vector form is given by $\vec{d} = \vec{a} - \left(\frac{\vec{a}\cdot\vec{b}}{|b|^2}\right)\vec{b}$

**MCQ:** $\vec{F} = (2\hat{i} - \hat{j} + 3\hat{k})N$, where $\vec{s} = (\hat{i} + 2\hat{j} + 5\hat{k})m$ then $\vec{F}\cdot\vec{s} =$ ______?

**EXPLANATION:**

$$\vec{F}\cdot\vec{s} = F_x s_x + F_y s_y + F_z s_z$$

$\Rightarrow \vec{F}\cdot\vec{s} = (2\times1) + (-1\times2) + (3\times5) \Rightarrow 2 - 2 + 15 \rightarrow \vec{F}\cdot\vec{s} = 15Nm$

**MCQ:** If $\vec{A} = -\hat{i} + \hat{j}$ and $\vec{B} = -\hat{j} + \hat{k}$ then find the $\theta$ between $\vec{A}'$ and $\vec{B}$______?

**EXPLANATION:**

$$|\vec{A}| = \sqrt{2} \text{ and } |\vec{B}| = \sqrt{2} \Rightarrow \vec{A}\cdot\vec{B} = -1$$

We know that:

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} \Rightarrow \text{used to find angle between two vectors}$$

$$\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{AB}$$

After putting the values the angle between the two vectors can be calculated.

**MCQ:** $\vec{A} = 2\hat{i} - 3\hat{j}$ now tell which vector will be perpendicular to $\vec{A}$______?

a) $3\hat{i} + 2\hat{j}$
b) $3\hat{i} - 2\hat{j}$ (correct)
c) $-2\hat{i} + 3\hat{j}$
d) $-2\hat{i} - 3\hat{j}$

**EXPLANATION:**

In solving these types of MCQ's first of all add all vectors the vector whose resultant come out to be zero will be perpendicular.

$\vec{A}\cdot\vec{B} = 0$ → that will be perpendicular → Let's check

a) $3\hat{i} + 2\hat{j} \rightarrow 6 + 6 = 12$
b) $3\hat{i} - 2\hat{j} \rightarrow 6 - 6 = 0$
c) $-2\hat{i} + 3\hat{j} \rightarrow -4 + 9 = 5$
d) $-2\hat{i} - 3\hat{j} \rightarrow -4 - 9 = -13$

**MCQ:** $\vec{A}\times\vec{B} = 0$ and $\vec{A}\cdot\vec{B} = 0$ then $\theta$ between tHem will be______?

a) 90°
b) 180°
c) 45°(correct)
d) 60°

**EXPLANATION:** $\vec{A}\times\vec{B} = \vec{A}\cdot\vec{B} \rightarrow AB\sin\theta = AB\cos\theta \rightarrow \sin\theta = \cos\theta$

$$\frac{\sin\theta}{\cos\theta} = 1 \rightarrow \tan\theta = 1 \rightarrow \theta = \tan^{-1}(1) \rightarrow \theta = 45°$$

So, on 45° the cross and dot product of two vectors become equal.

**SOME IMPORTANT POINTS:**

$$A_x = \frac{\vec{A}\cdot\vec{B}}{B} \Rightarrow \text{gives us component A along B}$$

$$B_x = \frac{\vec{A}\cdot\vec{B}}{A} \Rightarrow \text{gives us component of B along A}$$

**MCQ:** Vector a has magnitude 3, vector b has magnitude 4 and the angle between a and b is 60 What is the value of a.b ?

A: 3 B: 5 C: 6 D: 10.39

**SOLUTION:** *Use the formula* $a.b = |a| \times |b| \times \cos(\theta)$

*with* $|a| = 3, |b| = 4$ *and* $\theta = 60°$

*So* $a.b = 3 \times 4 \times \cos(60°) = 12 \times 0.5 = 6$

**MCQ:** Vector **a** has magnitude $3\sqrt{2}$, vector **b** has magnitude 5 and the angle between **a** and **b** is 135°. What is the value of **a.b** ?

A: 15 B: $15\sqrt{2}$ C: $15\sqrt{2}$ D: 15

**SOLUTION:** Use the formula $a.b = |a| \times |b| \times \cos(\theta)$

$$\text{with } |a| = 3\sqrt{2}, |b| = 5 \text{ and } \theta = 135°$$

So $a.b = 3\sqrt{2} \times 5 \times \cos(135^0) = 15\sqrt{2} \times \frac{-1}{\sqrt{2}} = -15$

## Vector product or Cross product of two vector

⇒ (Vector) x (Vector) = Vector.

**WE CAN CALCULATE THE CROSS PRODUCT THIS WAY:**

$$a \times b = |a|\,|b| \sin(\theta)\, n$$

⇒ |**a**| is the magnitude (length) of vector **a**

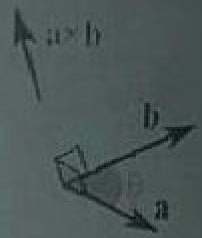
⇒ |**b**| is the magnitude (length) of vector **b**

⇒ θ is the angle between **a** and **b**

⇒ **n** is the unit vector at right angles to both **a** and **b**

⇒ **So, the length is**: the length of **a** times the length of **b** times the sine of the angle between **a** and **b**,

⇒ Then we multiply by the vector **n** to make sure it heads in the right **direction** (at right angles to both **a** and **b**).




⇒ $\vec{A} \times \vec{B} = AB \sin\theta \hat{n} \Rightarrow \vec{A} \times \vec{B} = \vec{C}\hat{n}$ → where C is the resultant vector

⇒ $\vec{A} \times \vec{B} = C$ →

so C is perpendicular to plane of A and B, or we can say that angle between A and C = 90°, or Angle between B and C = 90°.

Like; that Torque; $\tau = \vec{r} \times \vec{F}$ → angle between τ and $\vec{r}$ = 90°

⇒ ***Magnitude of a cross product is given by;*** $|\vec{A} \times \vec{B}| = AB \sin\theta$

⇒ if θ = 0° or 180°, sin 0° = sin 180° = 0 (cross product will be zero) ⇒ $\vec{A} \times \vec{B} = \vec{0} \Rightarrow |\vec{A} \times \vec{B}| = 0$

⇒ if θ = 90° (cross product will be maximum) ⇒ $\vec{A} \times \vec{B} = AB\hat{n} \Rightarrow |\vec{A} \times \vec{B}| = AB$

⇒ ***Cross product is Anti commutative:*** $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

BUT:

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}|$$

Also;

$$\vec{A} \times \vec{B} = \vec{C}$$
$$\vec{B} \times \vec{A} = -\vec{C}$$

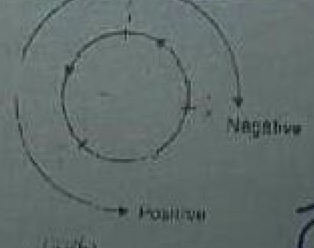
**And;**

$$(\vec{A} \times \vec{B}) + (\vec{B} \times \vec{A}) = \vec{0}$$
$$|(\vec{A} \times \vec{B}) + (\vec{B} \times \vec{A})| = 0$$

⇒ Cross product of a vector with itself is equal to; $\vec{a} \times \vec{a} = \vec{0}$.

⇒ ***Cross product of a unit vector with itself:*** $\hat{i} \times \hat{i} = 0$ and $\hat{j} \times \hat{j} = 0$ and $\hat{k} \times \hat{k} = 0$

(i) $\hat{i} \times \hat{j} = \hat{k}$ (ii) $\hat{j} \times \hat{k} = \hat{i}$ (iii) $\hat{k} \times \hat{i} = \hat{j}$ (iv) $\hat{i} \times \hat{k} = -\hat{j}$ (v) $\hat{k} \times \hat{j} = -\hat{i}$ (vi) $\hat{j} \times \hat{i} = -\hat{k}$




$\bar{c} = \frac{(a \times b)}{|a \times b|}$

⇒ The unit vector normal to $\vec{a}$ and $\vec{b}$ is $\vec{c} = \pm\frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$

⇒ Cross product in component form; If $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

**MCQ:** $\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k})m$ where $\vec{F} = (\hat{i} + 3\hat{j} + \hat{k})N$ tHen find $\tau$ = ______?

**EXPLANATION:** As we know that;

$$\tau = (r_yF_z - r_zF_y)\hat{i} + (r_zF_x - r_xF_z)\hat{j} + (r_xF_y - r_yF_x)$$

$$\tau = (1 - 9)\hat{i} + (3 - 2)\hat{j} + (6 - 1)\hat{k}$$

$$\tau = (-8)\hat{i} + (-1)\hat{j} + (5)\hat{k}$$

$$\tau = (-8\hat{i} + \hat{j} + 5\hat{k})Nm \rightarrow \text{vectorial form}$$

**NOTE THAT MAGNITUDE OF CROSS [RODUCT WILL GIVE US AREA OF PARALLELOGRAM.**

## MOST IMPORTANT MCQ Points:

- $(\vec{A} \times \vec{B}) = (\vec{B} \times \vec{A}) \Rightarrow$ tHen $\theta = 0°$ or $180°$
- $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \Rightarrow$ if $\theta$ is neither $0°$ nor $180°$
- $(\vec{A} \times \vec{B}) = \vec{A} \cdot \vec{B}$ if $\theta = 45°$
- $\vec{A} \times \vec{B} \neq \vec{A} \cdot \vec{B}$ but $|\vec{A} \times \vec{B}| = |\vec{A} \cdot \vec{B}|$
- $\vec{0} \cdot \vec{A} = 0$
- $\vec{0} \times \vec{A} = \vec{0}$

## Equilibrium

Equilibrium of a body is a state in which all the forces acting on the body are balanced (cancelled out), and the net force acting on the body is zero. The state of equilibrium is a very important concept to learn in physics. If the net resultant force acting on a body is zero, it means that the net acceleration of the body is also zero (from second law of motion).

Types of equilibrium of concurrent forces:

Generally, there are two types of equilibrium:

**1) Static equilibrium:** This is the type of equilibrium in which the resultant of all the forces acting on the body is zero, i.e. the net acceleration of the body is zero, and the velocity of body is also zero. It means that the body is at rest. So, if a body is at rest and the net acceleration of it is zero, it means the body is in static equilibrium.

**V=0** or $\omega = 0$
**a =0** 0r $\alpha=0$

Suppose a block is resting on a floor and two forces of 5N each are acting on it from either side. The forces would cancel each other out and hence the net force on the block would be zero. Since the block is at rest, it will be in static equilibrium.

**2) Dynamic equilibrium:** This is the type of equilibrium in which the resultant of all the forces acting on the body is zero, i.e. the net acceleration of the body is zero, but the velocity of body is not zero. It means that the body is moving with a constant velocity. So if the net force acting on the body is zero, and it is still moving with some constant velocity, the body is said to be in dynamic equilibrium.

**V=Constant $\Delta v=0$, a=0** $\omega$ **=constant** $\Delta\omega = 0, \alpha = 0$

A block attached to a spring and doing simple harmonic motion (S.H.M.) is a common example of dynamic equilibrium. At the mean position, the net force acting on the block is zero but the velocity of block is maximum, which means that the block is in dynamic equilibrium at that point.

(i) **Static translatory:** v=0 , a=0
(ii) **Static rotatory:** $\omega = 0$, $\alpha = 0$

(iii) **Dynamic translatory:** v= constant $\Delta v=0$, $a=0$
(iv) **Dynamic Rotatory:** $\omega$ =constant $\Delta\omega=0$, $\alpha=0$

## Some important Points and Tricks

- If $\vec{A}\times\vec{B}=\vec{C}$ and then $\vec{C}\perp\vec{A}$ as well as $\vec{B}$
- $|\vec{A}\times\vec{B}|=\vec{A}\cdot\vec{B}$ when $\theta=45^0$.
- The rectangular components can't have magnitude greater than vector itself [ $\cos\theta\le 1,\forall\theta$ ].
- If $\vec{A}_1+\vec{A}_2+\vec{A}_3+\ldots\ldots\ldots+\vec{A}_n=0$
  $\vec{A}_1=\vec{A}_2=\vec{A}_3=\ldots\ldots\ldots=\vec{A}_n$, then the adjacent vector can be inclined to each other at angle $\frac{2\pi}{9}$.
- If any two vectors are parallel or equal, then the scalar triple product is zero.
- The magnitude of $\vec{p}\times\vec{Q}$ can vary from maximum value $|\vec{p}|+|\vec{Q}|$ to minimum value $|\vec{p}|-|\vec{Q}|$
- For resultant of two vectors of equal magnitude.

| $\theta$ | Magnitude of Resultant |
|---|---|
| $60^0$ | $\sqrt{3}a$ |
| $90^0$ | $\sqrt{2}a$ |
| $120^0$ | $a$ |

where 'a' stands for magnitude of each vector.

- $\vec{a}=\lambda\vec{b}$, (a) $\vec{a}$ is parallel to $\vec{b}$, if $\lambda>0$; (b) $\vec{a}$ is antiparallel to $\vec{b}$, if $\lambda<0$
- When $F_1=F_2=F$ then R= F + F
- When R= 2F then $\theta=0^0$
- When R= $\sqrt{2}F\rightarrow\sqrt{2F^2}\rightarrow$ then $\theta=90^0$
- When R= 0 then $\theta=180^0$
- When R= F then $\theta=120^0$
- If A = B = C and R = C, then angle between A and B = $120^0$
- If $\vec{A}-\vec{B}=\vec{C}$ and $\vec{A}^2+\vec{B}^2=\vec{C}^2$ and $\vec{C}^2=\vec{A}^2+\vec{B}^2$ only possible on $\theta=90^0$.
- When $\vec{A}+\vec{p}=\vec{p}$ and $\vec{A}-\vec{B}=\vec{Q}$ when $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$ or $|\vec{P}|=|\vec{Q}|$ then $\theta=90^0$
- $\vec{A}+\vec{B}=\vec{A}-\vec{B}$ only when A=0 or B=0 or both zero
- When $B\neq0$ or $A\neq0$ then $\vec{A}+\vec{B}\neq\vec{A}-\vec{B}$.
- $\vec{A}+\vec{B}=\vec{p}$ and $\vec{A}+\vec{B}=\vec{Q}$ where $|\vec{A}|=|\vec{B}|$ and $|\vec{p}|=|\vec{Q}|$ then
  - $\Rightarrow$ Then $\theta$ between $\vec{A}$ and $\vec{P}=45^0$
  - $\Rightarrow$ $\theta$ between A and Q = $45^0$
  - $\Rightarrow$ Also $\theta$ between P and Q=$90^0$
- If $R=\vec{A}+\vec{B}$ when we increase any one vector n-times resultant also increases n-times then $\theta=120^0$. →R=A+B now if A=n A and B=n B then R= n R.
- If R= A + B where A ≠ B and R is perpendicular to smaller one then $\theta=120^0$.
- If $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$ then $\theta$ between $\vec{A}$ and $\vec{B}=90^0$.
- $\vec{A}\cdot\vec{B}=AB\cos\theta$ then $(\vec{A}\times\vec{B})=AB\sin\theta$
- $\vec{A}\cdot\vec{B}=AB\sin\theta$ then $(\vec{A}\times\vec{B})=AB\cos\theta$

## Some important forces

(1) **Concurrent Forces:** If line of action of force overlap at single point
Or Force act at a single point
(2) **Collinear Forces:** If line of action is same
$\Rightarrow$ All collinear forces are concurrent.

⇒ All concurrent are not collinear

(3) **Coplanar forces:** If a force lies in a single plane

(4) **Co initial forces:** Whose starting position are same or common

## How to find the resultant force acting on an object

⇒ First of all, let's answer the question: What is the resultant force? When an object is subject to several forces, the resultant force is the force that alone would produce the same effect as all those forces.

**How can the resultant force be found?**

⇒ Experiments show that when an object is subject to several forces $F_1$, $F_2$, ...., the resultant force is the vector sum of those forces. In symbols:

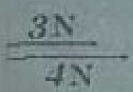
$R = F_1 + F_2 + \dots$

⇒ Therefore, it is not a mere sum of magnitudes of all the forces as you may think, but the sum of all the forces taken as vectors, which is more complex since vectors have both a magnitude and a direction that we need to consider when doing the sum.

⇒ In the following sections we will go over the process of finding the resultant force in different cases, starting from simple ones and moving to more complex ones.

1) Two forces acting in the same direction

Suppose we have an object subject to two forces that act in the same direction:




The resultant force will be in the same direction as the two forces, and have magnitude equal to the sum of the two magnitudes:




2) Two forces acting in opposite directions

Let's assume that an object is subject to two forces that act in opposite directions. In the case where the two forces are equal in magnitude:




The resultant force will be **zero** since the two opposite forces **cancel each other out**.

3) On the other hand, if the two forces are not equal in magnitude:




The resultant force will be in the **same direction as the larger force** (the 5N force in the example), and have the magnitude equal to the **difference between the magnitudes of the two forces** (in our example that would be 2N).




4) More than 2 forces parallel to one another

Let's now consider more than two forces




To find the resultant force, we first need to sum all the forces that go in one direction, and then all the forces that go in the other direction:



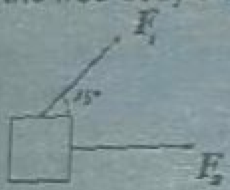

We have now two forces that are in opposite directions, which is a case we already know how to solve: the resultant force is in the direction of the larger force (the 11N force) and has magnitude equal to the difference between the two magnitudes (4N).




5) Two forces that are not parallel

⇨ In the previous cases, we had forces that were parallel to one another, but let's now consider a more complex case with two forces that are not parallel.
Suppose we have a block subject to two forces, which we'll indicate with $F_1$ and $F_2$.

⇨ $F_1$ has magnitude 50N and is applied at a 45° angle, whereas $F_2$ has magnitude 60N and is applied horizontally, as shown in the free-body diagram below:

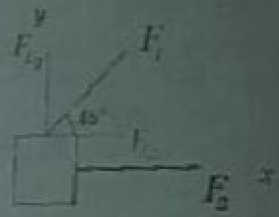



So how do we find the resultant force R in this case?
The first step is to **draw coordinate axes on our free-body diagram.**

⇨ Since one of the two forces is horizontal, it is convenient to choose the **x** axis in the horizontal direction, the **y** axis in the vertical direction, and place the **origin** right in the center of our object:




Our next step is to **<u>determine the x and y components</u> of the individual forces.**




$$F_{1x} = F_1 \cos 45°, F_{2x} = F_2,$$
$$F_{1y} = F_1 \sin 45°, F_{2y} = 0$$

## Summary

To find the resultant force acting on an object, follow these steps:

1. Draw a free-body diagram of the object
2. Draw coordinate axes on the free-body diagram
3. Decompose the forces acting on the object into x and y components
4. Calculate the x and y component of the resultant force by adding the x and y components of all forces
5. Finally, find the magnitude and direction of the resultant force by using its x and y component

## Formula sheet

| S/N | QUANTITY | EQUATION |
|---|---|---|
| 1) | *vector* | *magnitude + direction* |
| 2) | *Scalar* | *Magnitude* |
| 3) | *Unit vector* | $unit\ vector = \frac{vector}{Magnitude} = \frac{\vec{A}}{\lvert\vec{A}\rvert}$ |
| 4) | *Position vector* | Vector:<br>$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$<br>Magnitude:<br>$\lvert\vec{r}\rvert = \sqrt{x^2 + y^2 + z^2}$<br>Direction:<br>$tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\frac{y}{x}$ |
| 5) | *Displacement vector/Free vector* | Vector:<br>$\Delta\vec{r} = r_2 - r_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$<br>Magnitude:<br>$\lvert\Delta\vec{r}\rvert = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ |
| 6) | *Addition of vector* | Vector:<br>$\vec{R} = \vec{A} + \vec{B}$<br>Magnitude:<br>$\lvert\vec{R}\rvert = \sqrt{A^2 + B^2 + 2ABcos\theta}$<br>$\lvert\vec{R}\rvert \leq A + B$<br>$\lvert\vec{R}\rvert = A + B\ (\theta = 0°)$<br>$\lvert\vec{R}\rvert < A + B\ (for\ angle\ other\ tha\ 0°)$<br>$\lvert\vec{R}\rvert = A - B\ or\ B - A\ (\theta = 180°)$ |
| 7) | *Resultant of vector* | $Min \leq R \leq Max\ possible\ sum$ |
| 8) | *Resolution of vector* | For x-component:<br>$F_x = Fcos\theta$<br>$\vec{F_x} = Fcos\theta\hat{i}(vectorial\ form)$<br>For y-component:<br>$F_y = F\ sin\theta$<br>$\vec{F_y} = F\ sin\theta\hat{j}\ (vectorial\ form)$<br>For vector F:<br>$\vec{F} = F_x\hat{i} + F_y\hat{j}$<br>Magnitude of F:<br>$F = \sqrt{F_x^2 + F_y^2 + +F_z^2}$<br>For direction of $\vec{F}$:<br>$tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1}\frac{F_y}{F_x}$ |
| 9) | Addition of vector w.r.t component: | For horizontal $R_x$:<br>$\vec{R_x} = \vec{A_x} + \vec{B_x}$<br>For vertical $R_y$:<br>$R_y = \vec{A_y} + \vec{B_y}$<br>Magnitude of R:<br>By Pythagoras theorem:<br>$(hyp)^2 = (Base)^2 + (perpendicular)^2$ |

| | | |
|---|---|---|
| | | $R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$<br>or $R = \sqrt{R_x^2 + R_y^2}$<br>For direction:<br>$\tan\theta = \frac{R_y/\,or(A_y + B_y)}{R_x/or(A_x + B_x)}$<br>$\theta = \tan^{-1}\left(\frac{(A_y + B_y)}{(A_x + B_x)}\right)$ |
| 10) | Quadrants | $1st\ Quadrant \rightarrow \phi = \theta$<br>$2nd\ Quadrant \rightarrow \theta = 180^\circ - \phi$<br>$3rd\ Quadrant \rightarrow \theta = 180^\circ + \phi$<br>$4th\ Quadrant \rightarrow \theta = 360^\circ - \phi$ |
| 11) | Scalar product | $\vec{A}.\vec{B} = AB\cos\theta$<br>Angle:<br>$\cos\theta = \frac{\vec{A}.\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$<br>Components:<br>$A_x = \frac{\vec{A}.\vec{B}}{B}$<br>$B_x = \frac{\vec{A}.\vec{B}}{A}$ |
| 12) | dot product in component form | $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$ |
| 13) | Vector product | $\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$ |
| 14) | vector product in component form | $\vec{A} \times \vec{B} =$<br>$(A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j}$<br>$+ (A_xB_y - A_yB_x)\hat{k}$ |
| 15) | Torque | $\vec{\tau} = \vec{r} \times \vec{f}$<br>$\vec{\tau} = \vec{r}f\sin\theta$ |
| 16) | Equilibrium | $a = 0, \alpha = 0$<br>$v = 0, \omega = 0$<br>Or<br>$v = constant, \omega = constant$<br>First condition of Equilibrium:<br>$\sum \vec{F} = 0$<br>2nd Condition of Equilibrium:<br>$\sum \vec{\tau} = 0$ |

## Some Important Points and Tricks:

1) All physical quantities having direction are not vectors. For example, the electric current possesses direction but it is not a vector.
2) A vector can have only two rectangular components in plane and only three rectangular components in space.
3) A vector can have any number, even infinite components. (Minimum 2 components).
4) Following quantities are neither vectors nor scalars: Relative density, density, viscosity, frequency, pressure, stress, strain, modulus of elasticity, poison's

ratio, moment of inertia, specific heat, latent heat, spring constant loudness, resistance, conductance, reactance, impedance, permittivity, dielectric constant, permeability, susceptibility, refractive index, focal length, power of lens, Boltzmann constant, Stefan's constant, Gas constant, Gravitational constant, Rydberg constant, Planck's constant etc.

5) Scalars are added, subtracted or divided algebraically
6) Vectors are added and subtracted geometrically
7) Division of vectors is not allowed as directions cannot be divided
8) Unit vector gives the direction of vector.
9) Magnitude of unit vector is 1.
10) Unit vector has no unit. For example, velocity of an object is 5 $ms^{-1}$ due East

$$i.e\ \vec{v} = 5ms^{-1}\ due\ east$$

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{5ms^{-1}(East)}{5ms^{-1}} \Rightarrow East$$

*So unit vector $\hat{v}$ has no unit as East is not a physical quantity.*

11) Unit vector has no dimensions.
12) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
13) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$
14) $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$
15) $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$
16) $\vec{A}\times\vec{A} = \vec{0}$. Also $\vec{A}-\vec{A} = \vec{0}$ but $\vec{A}\times\vec{A} \neq \vec{A}-\vec{A}$ because $\vec{A}\times\vec{A} \perp \vec{A}$ and $\vec{A}-\vec{A}$ is collinear with $\vec{A}$.
17) Multiplication of a vector with $-1$ reverses its direction.
    1. $if\ \vec{A} = \vec{B}, then\ A = B\ and\ \hat{A} = \hat{B}$
    2. $if\ \vec{A} + \vec{B} = \vec{0}, then\ A = B\ but\ \hat{A} = -\hat{B}$
18) Two vectors are perpendicular to each other if $\vec{A}.\vec{B} = \vec{0}$
19) Two vectors are parallel to each other if $\vec{A}\times\vec{B} = \vec{0}$
20) Displacement, velocity, linear momentum and force are polar vectors.
21) Angular velocity, angular acceleration, torque and angular momentum are axial vectors.
22) Division with a vector is not defined because it is not possible to divide with a direction.
23) Distance covered is always positive quantity.
24) The components of a vectors can have magnitude than that of the vector itself
25) The rectangular components cannot have magnitude greater than that of the vector itself.
26) When we multiply a vector with 0 the product becomes a null vector.
27) The resultant of two vectors of unequal magnitude can never be a null vector
28) Three vectors not lying in a plane can never add up to give a null vector
29) A quantity having magnitude and direction is not necessarily a vector. For example, time and electric current. These quantities have magnitude and direction but they are scalar. This is because they do not obey the laws of vector addition.
30) A physical quantity which has different values in different directions is called a tensor. For example : Moment of inertia has different values in different directions. Hence moment of inertia is a tensor. Other examples of tensor are refractive index, stress, strain, density etc.
31) The magnitude of rectangular components of a vector is always less than the magnitude of the vector
32) $if\ \vec{A} = \vec{B}$ , then $A_x = B_x, A_y = B_y, A_z = B_z$
33) $if\ \vec{A} = \vec{B} = \vec{C}$ or if $\vec{A} + \vec{B} + \vec{C} = \vec{0}$, then $\vec{A}\ \vec{B}\ and\ \vec{C}$ lies in one plane.
34) $if\ \vec{A}\times\vec{B} = \vec{C}, then\ \vec{C} is \perp too\ \vec{A}\ as\ well\ as\ \vec{B}$.
35) $if\ |\vec{A}\times\vec{B}| = |\vec{A}-\vec{B}|$, then angle between $\vec{A}\ and\ \vec{B}$ is 90°
36) Resultant of two vectors will be maximum when $\theta = 0°$ i.e. vectors are parallel

$$R_{max} = \sqrt{P^2 + Q^2 + 2PQ\cos 0°} = |P+Q|$$

37) Resultant of two vectors will be minimum when $\theta = 180°$ i.e. vectors are anti-parallel.

$$R_{min} = \sqrt{P^2 + Q^2 + 2PQ\cos 180°} = |P-Q|$$

Thus, minimum value of the resultant of two vectors is equal to the difference of their magnitude.

38) Thus, maximum value of the resultant of two vectors is equal to the sum of their magnitude.

When the magnitudes of two vectors are unequal, then

$$R_{min} = P - Q \neq 0$$
$$|\because |\vec{P}| \neq |\vec{Q}||$$

Thus, two vectors $\vec{P}$ *and* $\vec{Q}$ having different magnitudes can never be combined to give zero resultant. From here, we conclude that the minimum number of vectors of unequal magnitude whose resultant can be zero is three. On the other hand, the minimum number of vectors of equal magnitude whose resultant can be zero is two

39) Angle between two vectors $\vec{A}$ *and* $\vec{B}$ *is given by*

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$$

40) Projection of a vector $\vec{A}$ in the direction of vector $\vec{B}$

$$= \frac{\vec{A}.\vec{B}}{|\vec{B}|}$$

41) Projection of a vector $\vec{B}$ in the direction of vector $\vec{A}$

$$= \frac{\vec{A}.\vec{B}}{|\vec{A}|}$$

42) If vectors $\vec{A}$, $\vec{B}$ *and* $\vec{C}$ are represented by three sides ab, bc and ca respectively taken in a order, then

$$\frac{|\vec{A}|}{ab} = \frac{|\vec{B}|}{bc} = \frac{|\vec{C}|}{ca}$$

43) The vectors $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the coordinate axes at an angle of 54.74 degrees.

44) If $\vec{A} \pm \vec{B} = \vec{C}$, *then* $\vec{A}.\vec{B} \times \vec{C} = 0$

45) If $\vec{A}.\vec{B} \times \vec{C} = 0$ then $\vec{A}.\vec{B}$ *and* $\vec{C}$ *are coplanar*

46) If angle between $\vec{A}$ *and* $\vec{B}$ *is* $45°$, *then* $\vec{A}.\vec{B} = |\vec{A} \times \vec{B}|$

47) If $\vec{A_1} + \vec{A_2} + \vec{A_3} + \dots\dots\dots + \vec{A_n} = \vec{0}$ *and* $A_1 = A_2 = A_3 = \dots\dots\dots = A_n$ then the adjacent vector are inclined to each other at angle $\frac{2\pi}{n}$.

48) If $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$ then the angle between $\vec{A}$ *and* $\vec{B}$ *is* $90°$. Also A, B and C can have the following values

i. $A = 3, B = 4, C = 5$
ii. $A = 5, B = 12, C = 13$
iii. $A = 8, B = 15, C = 17$

49) Minimum forces of equal magnitude for equilibrium they are two, they must be:

- ⇒ Opposite
- ⇒ Collinear
- ⇒ Concurrent
- ⇒ Coplanar
- ⇒ If one of them is not obeyed then they will be not at equilibrium

50) Maximum forces of equal magnitude for equilibrium are infinite.

51) Maximum unequal forces for equilibrium are three but they must be;

- ⇒ Concurrent
- ⇒ Non-collinear
- ⇒ Coplanar

52) Maximum number of unequal forces for equilibrium are infinite.

53) Angle between adjacent forces of equal magnitude for equilibrium→ $\theta = \frac{360^0}{n}$

→Where n represent number of forces.

54) Three forces acting on a body of equal magnitude if we add them by head to tail rule if they form 60° and form equilateral triangle then they will be at equilibrium.

55) If we apply force equal in magnitude to resultant but opposite in direction then the body will attain equilibrium state.

56) **Important:** If a body were in equilibrium if n-forces act on a body if we remove 1 force then the resultant force acting on a body will have same magnitude as that force which we remove but opposite in direction.

57) Torque: Torque= $\vec{r} \times \vec{f}$ → if position changed direction will also changed

$\vec{r} \times \vec{f}$→ Anticlockwise→ (+ve)

$\vec{f} \times \vec{r}$→ Clockwise→ (-ve)

58) *Couple forces:* Two forces of equal magnitude, Antiparallel, Non collinear Non concurrent, Coplanar forces → These forces produce angular acceleration and satisfy 1st condition of equilibrium

$\vec{F_1} = -\vec{F_2}$ → $F_1$ [illegible] 0 so→ a=0

59) *If forces are not equal use the formula Torque(net) =* $\tau_{net} = \tau = \tau_1 + \tau_2 + \tau_3 = r_1 f_1 + r_2 f_2$

60) If the forces are equal then use this formula: $\tau = LF$ → Where L is the couple arm → The perpendicular distance between couple forces.

61) If two vectors are at right angles to each other than there scalar product will be zero.

62) $\vec{A} \times \vec{B}$ is perpendicular to both $\vec{A} + \vec{B}$ as well as $\vec{A} - \vec{B}$ and $\vec{A} + \vec{B}$ as well as $\vec{A} - \vec{B}$ lie in the plane containing $\vec{A}$ and $\vec{B}$, but $\vec{A} \times \vec{B}$ lies perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

63) Angle between $\hat{i} + \hat{j}$ and $\hat{i}$ is $45°$.

64) $\hat{i} + \hat{j}$ is equally inclined to both $\hat{i}$ and $\hat{j}$ and the angle between $\hat{i}$ and $\hat{j}$ is $90°$.

65) If $\theta$ be the angle between $\vec{A}$ and $\vec{B}$, then

$$\tan\theta = \frac{\vec{A} \times \vec{B}}{\vec{A}.\vec{B}}$$

66) $\vec{A} \times \vec{B}$ is perpendicular to $\vec{A}.\vec{B}$.

67) If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ is $90°$.

68) $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

69) $\vec{A}.\vec{B} = \vec{B}.\vec{A}$.

70) Vector product of two vectors is an axial vector.

71) Minimum number of non-equal vectors in a plane required to give zero resultant is three.

72) If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$ then they must lie in one plane.

73) Relative velocity of A w.r.t. B is greater than the velocity of either, when they are moving in opposite directions.

74) Vector addition of two vectors $\vec{A}$ and $\vec{B}$ is commutative.

75) Cross product of vectors is commutative.

76) $\tau = \vec{r} \times \vec{F}$ and $\tau \neq \vec{F} \times \vec{r}$

77) A negative acceleration of a body is associated with a slowing down of a body.

78) A physical quantity cannot be called as a vector if its magnitude is zero.

79) The sum of two vectors can be zero.

80) The vector cancel each other, when they are equal and opposite.

81) Two vectors are said to be like vectors if they have same direction but different magnitude.

82) Vector quantities do not have specific direction.

83) The scalar product of two vectors can be zero.

84) If two vectors are perpendicular to each other, their scalar product will be zero.

85) Multiplying any vector by an scalar is a meaningful operations.

86) A null vector is a vector whose magnitude is zero and direction is arbitrary.

87) A null vector does not exist.

88) If dot product and cross product of $\vec{A}$ and $\vec{B}$ are zero, it implies that one of the vector $\vec{A}$ and $\vec{B}$ must be a null vector.

89) Null vector is a vector with zero magnitude.

90) The cross product of a vector with itself is a null vector.

91) The cross-product of two vectors results in a vector quantity.

92) The minimum number of non coplanar vectors whose sum can be zero, is four.

93) The resultant of two vectors of unequal magnitude can be zero.

94) If $\vec{A}.\vec{B} = \vec{B}.\vec{C}$ then $\vec{A}$ may not always be equal to $\vec{C}$.

95) The dot product of two vectors involves cosine of the angle between the two vectors.

96) $(\vec{A} + \vec{B}) \neq (\vec{B} + \vec{A})$

## MULTIPLE CHOICE QUESTIONS

1) As the angle between two concurrent forces increases then the magnitude of there resultant___?
(a) Increases (b) Decreases
(c) Increases and then decreases
(c) Decreases and then increases

2) A girl walks 2m north, 4m west and 2m south her final displacement is________?
(a) 4m east (b) 4m west
(c) 2m north (d) 2m south

3) What is not correct to two forces that give rise to a couple________?
(a) Acts in opposite direction ?
(b) Both acts at a same point
(c) Both acts on the same body
(d) Both have same magnitude

4) The vector projection of a vector $3\hat{i} + 4\hat{k}$ on y-axis is____________?
(a) 5 (b) 4 (c) 3 (d) Zero

5) Position of a particle in a rectangular-co-ordinate system is (3, 2, 5 ). Then its position vector will be________?
(a) $3\hat{i} + 5\hat{j} + 2\hat{k}$ (b) $3\hat{i} + 2\hat{j} + 5\hat{k}$ (c) $5\hat{i} + 3\hat{j} + 2\hat{k}$ (d) None

6) If a particle moves from point P (2,3,5) to point Q (3,4,5). Its displacement vector be____?
(a) $\hat{i} + \hat{j} + 10\hat{k}$ (b) $\hat{i} + \hat{j} + 5\hat{k}$
(c) $\hat{i} + \hat{j}$ (c) $2\hat{i} + 4\hat{j} + 6\hat{k}$

7) A force of 5 N acts on a particle along a direction making an angle of 60° with vertical. Its vertical component be
(a) 10 N (b) 3 N (c) 4 N (d) 2.5 N

8) If $A = 3\hat{i} + 4\hat{j}$ and $B = 7\hat{i} + 24\hat{j}$ the vector having the same magnitude as B and parallel to A is________?
(a) $5\hat{i} - 20\hat{j}$ (b) $15\hat{i} + 10\hat{j}$
(c) $20\hat{i} + 15\hat{j}$ (d) $15\hat{i} + 20\hat{j}$

9) Vector $\vec{A}$ makes equal angles with x, y and z axis. Value of its components (in terms of magnitude of $\vec{A}$) will be______?
(a) $\frac{A}{\sqrt{3}}$ (b) $\frac{A}{\sqrt{2}}$ (c) $\sqrt{3}A$ (d) $\frac{\sqrt{3}}{A}$

10) The vector that must be added to the vector $\hat{i} - 3\hat{j} + 2\hat{k}$ and $3\hat{i} + 6\hat{j} + 7\hat{k}$ so that the resultant vector is a unit vector along the y-axis is_________?
(a) $4\hat{i} + 2\hat{j} + 5\hat{k}$ (b) $-4\hat{i} - 2\hat{j} + 5\hat{k}$
(c) $3\hat{i} + 4\hat{j} + 5\hat{k}$ (d) Null vector

11) How many minimum numbers of coplanar vectors having different magnitudes can be added to give zero resultant_______?
(a) 2 (b) 3 (c) 4 (d) 5

12) A hall has the dimensions 10 m x 12 m x 14 m. A fly starting at of corner ends up at a diametrically opposite corner. What is the magnitude of its displacement
(a) 17 m (b) 26 m?
(c) 36 m (d) 20 m

13) 100 coplanar forces each equal to 10 N act on a body. Each force makes angle $\pi / 50$ with the preceding force. What is the resultant of the forces______?
(a) 1000 N (b) 500 N
(c) 250 N (d) Zero

14) The magnitude of a given vector with end points (4, − 4, 0) and ( 2, −2, 0) must be?
(a) 6 (b) $5\sqrt{2}$ (c) 4 (d) $2\sqrt{10}$

15) The expression $(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j})$ is a _______?
(a) Unit vector b) Null vector
(c) Vector of magnitude $\sqrt{2}$ (d) Scalar

16) Given vector $\vec{A} = 2\hat{i} + 3\hat{j}$ the angle between $\vec{A}$ and y-axis is_____?
(a) $\tan^{-1} 3/2$ (b) $\tan^{-1} 2/3$
(c) $\sin^{-1} 2/3$ (d) $\cos^{-1} 2/3$

17) The unit vector along $\hat{i} + \hat{j}$ is_____?
(a) $\hat{k}$ (b) $\hat{i} + \hat{j}$ (c) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i}+\hat{j}}{2}$

18) A vector is represented by $3\hat{i} + \hat{j} + 2\hat{k}$ its length in xy plane is_____?
(a) 2 (b) $\sqrt{14}$ (c) $\sqrt{10}$ (d) $\sqrt{5}$

19) Five equal forces of 10 N each are applied at one point and all are lying in one plane. If the angles between them are equal, the resultant force will be?
(a) Zero (b) 10 N (c) 20 N (d) 102N

20) The angle made by the vector $\vec{A} = \hat{i} + \hat{j}$ with x- axis is_____?
(a) 90° (b) 45° (c) 22.5° (d) 30°

21) Any vector in an arbitrary direction can always be replaced by two (or three)_______?
(a) Parallel vectors which have the original vector as their resultant
(b) Mutually perpendicular vectors which have the original vector as their resultant
(c) Arbitrary vectors which have the original vector as their resultant
(d) It is not possible to resolve a vector

22) If $\vec{P} = \vec{Q}$ then which of the following is NOT correct_________?
(a) $\hat{P} = \hat{Q}$ (b) $|\vec{P}| = |\vec{Q}|$
(c) $P\hat{Q} = Q\hat{P}$ (d) $\vec{P} + \vec{Q} = \hat{P} + \hat{Q}$

23) The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity of the particle is
(a) Parallel to the position vector
(b) Perpendicular to the position vector
(c) Directed towards the origin

(d) Directed away from the origin

24) Two vectors A and B are such that |C|=|A| + |B| If θ is the angle between positive direction of vector A and vector B then the correct statement is?
a) θ=π b)2π/3 c) θ=0 d) θ=π/2

25) A particle starting from the origin (0,0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are (√3, 3). The path of the particle makes with the x-axis an angle of
a) 45° b) 60° c)0° d)30°

26) If vector A and B are perpendicular to each other than value of α is ...,
$$\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$$
$$\vec{B} = -4\hat{\imath} + 4\hat{\jmath} + \alpha\hat{k}$$
a)1/2 b)-1/2 c)1 d)-1

27) The vector sum of two forces is perpendicular to their vector differences. In that case, the forces...
a) cannot predicted
b) are equal to each other
c) are equal to each other in magnitude
d) are not equal to each other in magnitude

28) If a unit vector is represented by $0.5\hat{\imath} + 0.8\hat{\jmath} + c\hat{k}$ then the value of 'c' is______?
(a) 1 (b) $\sqrt{0.11}$ (c) $\sqrt{0.01}$ (d) $\sqrt{0\cdot 39}$

29) A boy walks uniformly along the sides of a rectangular park of size 400 m× 300 m, starting from one corner to the other corner diagonally opposite. Which of the following statement is incorrect
(a) He has travelled a distance of 700 m
(b) His displacement is 700 m
(c) His displacement is 500 m
(d) His velocity is not uniform throughout the walk

30) The unit vector parallel to the resultant of the vectors $\vec{A} = 4\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ and $\vec{B} = -\hat{\imath} + 3\hat{\jmath} - 8\hat{k}$_______?
(a) $\frac{1}{7}(3\hat{\imath} + 6\hat{\jmath} - 2\hat{k})$ (b) $\frac{1}{7}(3\hat{\imath} + 6\hat{\jmath} + 2\hat{k})$
(c) $\frac{1}{49}(3\hat{\imath} + 6\hat{\jmath} - 2\hat{k})$ (d) $\frac{1}{49}(3\hat{\imath} - 6\hat{\jmath} + 2\hat{k})$

31) Surface area is _________?
(a) Scalar (b) Vector
(c) Neither scalar nor vector
(d) Both scalar and vector

32) The angle between the two vectors $\vec{A} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{B} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ is________?
(a) 60° (b) Zero (c) 90° (d) None

33) The component of vector $A = 2\hat{\imath} + 3\hat{\jmath}$ along the vector $\hat{\imath} + \hat{\jmath}$ is______
(a) $\frac{5}{\sqrt{2}}$ (b) $10\sqrt{2}$ (c) $5\sqrt{2}$ (d) 5

34) The angle between the two vectors $\vec{A} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{B} = 3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ will be_______?
(a) 90° (b) 0° (c) 60° (d) 45°

35) There are two force vectors, one of 5 N and other of 12 N at what angle the two vectors be added to get resultant vector of 17 N, 7 N and 13 N respectively_____?
(a) 0°, 180° and 90° (b) 0°, 90° and 180°
(c) 0°, 90° and 90° (d) 180°, 0° and 90°

36) Q) If $\vec{A} = 4\hat{\imath} - 3\hat{\jmath}$ and $\vec{B} = 6\hat{\imath} + 8\hat{\jmath}$ then magnitude and directon of $\vec{A} + \vec{B}$ will be________?
(a) 5, tan$^{-1}$ (3/4) (b) 5√5, tan$^{-1}$ (1/2)
(c) 10, tan$^{-1}$(5) (d) 25, tan$^{-1}$ (3/4)

37) $\vec{A} = 2\hat{\imath} + \hat{\jmath}, \vec{B} = 3\hat{\jmath} - \hat{k}, \vec{C} = 6\hat{\imath} - 2\hat{k}$ then value of $\vec{A} - 2\vec{B} + 3\vec{C}$_______?
(a) $20\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$ (b) $20\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$
(c) $20\hat{\imath} - 12\hat{\jmath} + 4\hat{k}$ (d) $20\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$

38) Two forces, each of magnitude F have a resultant of the same magnitude F. The angle between the two forces is___
(a) 45° (b) 120° (c) 150° (d) 60°

39) For the resultant of the two vectors to be maximum, what must be the angle between them_______?
(a) 0° (b) 60° (c) 90° (d) 180°

40) A particle is simultaneously acted by two forces equal to 4 N and 3N. The net force on the particle is__________?
(a) 7 N (b) 5 N (c) 1 N (d) between 1N and 7N

41) Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane, another vector $\vec{C}$ lies outside this plane, then the resultant of these three vectors i.e., $\vec{A} + \vec{B} +$
(a) Can be zero (b) Cannot be zero
(c) Lies in the plane containing $\vec{A} + \vec{B}$
(d) Lies in the plane containing $\vec{C}$

42) If the resultant of the two forces has a magnitude smaller than the magnitude of larger force, the two forces must be_?
(a) Different both in magnitude and direction

(b) Mutually perpendicular to one another
(c) Possess extremely small magnitude
(d) Point in opposite directions

43) Forces $F_1$ and $F_2$ act on a point mass in two mutually perpendicular directions. The resultant force on the point mass will be______?
(a) $F_1 + F_2$ (b) $F_1 - F_2$
(c) $\sqrt{F_1^2 + F_2^2}$ (d) $\sqrt{F_1 - F_2}$

44) If $|\vec{A} - \vec{B}| = |\vec{A}| = |\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ is__________?
(a) 60° (b) 0° (c) 120° (d) 90°

45) Let the angle between two nonzero vectors $\vec{A}$ and $\vec{B}$ be 120° and resultant be $\vec{C}$__________?
(a) $\vec{C}$ must be equal to $|\vec{A} - \vec{B}|$
(b) $\vec{C}$ must be less that $|\vec{A} - \vec{B}|$
(c) $\vec{C}$ muat be greater than $|\vec{A} - \vec{B}|$
(d) $\vec{C}$ may be equal to $|\vec{A} - \vec{B}|$

46) The magnitude of vector $\vec{A}, \vec{B}$ and $\vec{C}$ are respectively 12, 5 and 13 units and $\vec{A} + \vec{B} = \vec{C}$ then the angle between $\vec{A}$ and $\vec{B}$ is________?
(a) 0 (b) $\pi$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

47) A particle has displacement of 12 m towards east and 5 m towards north then 6 m vertically upward. The sum of these displacements is__________?
(a) 12 (b) 10.04 m (c) 14.31 m (d) None

48) The three vectors $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}, \vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ form_______?
(a) An equilateral triangle
(b) Isosceles triangle
(c) A right-angled triangle(d) No triangle

49) Let $\vec{C} = \vec{A} + \vec{B}$ then___________?
(a) $\vec{C}$ is always greater then $\vec{A}$
(b) It is possible to have $|\vec{C}| < |\vec{A}|$ and $|\vec{C}| < |\vec{B}|$
(c) C is always equal to A + B
(d) C is never equal to A + B

50) Following sets of three forces act on a body. Whose resultant cannot be zero
(a) 10, 10, 10 (b) 10, 10, 20
(c) 10, 20, 23 (d) 10, 20, 40

51) When three forces of 50 N, 30 N and 15 N act on a body, then the body is__________?
(a) At rest
(b) Moving with a uniform velocity
(c) In equilibrium
(d) Moving with an acceleration

52) The sum of two forces acting at a point is 16 N. If the resultant force is 8 N and its direction is perpendicular to minimum force then the forces are ________?
(a) 6 N and 10 N (b) 8 N and 8 N
(c) 4 N and 12 N (d) 2 N and 14 N

53) If vectors P, Q and R have magnitude 5, 12 and 13 units and $\vec{P} + \vec{Q} = \vec{R}$, the angle between $\vec{Q}$ and $\vec{R}$ is__________?
(a) $\cos^{-1}\frac{5}{12}$ (b) $\cos^{-1}\frac{5}{13}$
(c) $\cos^{-1}\frac{12}{13}$ (d) $\cos^{-1}\frac{7}{13}$

54) The resultant of two vectors A and B is perpendicular to the vector A and its magnitude is equal to half the magnitude of vector B. The angle between A and B is__________?
(a) 120° (b) 150° (c) 135° (d) None

55) What is the angle between $\vec{P}$ and the resultant of $(\vec{P} + \vec{Q})$ and $(\vec{P} - \vec{Q})$_______?
(a) zero (b) $\tan^{-1}\frac{P}{Q}$
(c) $\tan^{-1}\frac{Q}{P}$ (d) $\tan^{-1}\frac{P-Q}{P+Q}$

56) The resultant of $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$. what is the angle between $\vec{P}$ and $\vec{Q}$___________?
(a) $\cos^{-1}\frac{P}{Q}$ (b) $\cos^{-1}-\frac{P}{Q}$
(c) $\sin^{-1}\frac{P}{Q}$ (d) $\sin^{-1}-\frac{P}{Q}$

57) Maximum and minimum magnitudes of the resultant of two vectors of magnitudes P and Q are in the ratio 3: 1. Which of the following relations is true___________?
(a) P = 2Q (b) P = Q
(c) PQ = 1 (d) None of these

58) The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If Q is doubled, the new resultant is perpendicular to P. Then R equals
(a) P (b) (P+Q)
(c) Q (d) (P-Q)

59) Two forces, $F_1$ and $F_2$ are acting on a body. One force is double that of the other force and the resultant is equal to

the greater force. Then the angle between the two forces is________?
(a) $\cos^{-1}\left(\frac{1}{2}\right)$ (b) $\cos^{-1}\left(\frac{-1}{2}\right)$
(c) $\cos^{-1}\left(\frac{-1}{4}\right)$ (d) $\cos^{-1}\left(\frac{1}{4}\right)$

60) Given that $\vec{A}+\vec{B}=\vec{C}$ and that $\vec{C}$ is ⊥ to $\vec{A}$. Further if $|\vec{A}|=|\vec{C}|$, then what is the angle between $\vec{A}$ and $\vec{B}$________?
(a) $\frac{\pi}{4}$ radian (b) $\frac{\pi}{2}$ radian
(c) $\frac{3\pi}{4}$ radian (d) $\pi$ radian

61) A body is at rest under the action of three forces, two of which are $\overrightarrow{F_1}=4\hat{i}$, and $\overrightarrow{F_2}=6\hat{j}$ the third force is__________?
(a) $4\hat{i}+6\hat{j}$ (b) $4\hat{i}-6\hat{j}$
(c) $-4\hat{i}+6\hat{j}$ (d) $-4\hat{i}-6\hat{j}$

62) A body moves due East with velocity 20 km/hour and then due North with velocity 15 km/hour. The resultant velocity
(a) 5 km/hour (b) 15 km/hour
(c) 20 km/hour (d) 25 km/hour

63) The magnitudes of vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are 3, 4 and 5 units respectively. If $\vec{A}+\vec{B}=\vec{C}$, the angle between $\vec{A}$ and $\vec{B}$ is__?
(a) $\frac{\pi}{2}$ (b) $\cos^{-1}0.6$
(c) $\tan^{-1}\left(\frac{7}{5}\right)$ (d) $\frac{\pi}{4}$

64) Two forces $\overrightarrow{F_1}=5\hat{i}+10\hat{j}-20\hat{k}$ and $\overrightarrow{F_2}=10\hat{i}-5\hat{j}-15\hat{k}$ acts on a single point. The angle between $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ is nearly______?
(a) 30° (b) 45° (c) 60° (d) 90°

65) Which pair of the following forces will never give resultant force of 2 N?
(a) 2 N and 2 N (b) 1 N and 1 N
(c) 1 N and 3 N (d) 1 N and 4 N

66) Three concurrent forces of the same magnitude are in equilibrium. What is the angle between the forces? Also name the triangle formed by the forces as sides______________?
(a) 60° equilateral triangle
(b) 120° equilateral triangle
(c) 120°, 30°, 30° an isosceles triangle
(d) 120° an obtuse angled triangle

67) If $|\vec{A}+\vec{B}|=|\vec{A}|+|\vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ will be__________?
(a) 90° (b) 120°
(c) 0° (d) 60°

68) The vector sum of two forces is perpendicular to their vector differences. In that case, the forces_____?
(a) Are equal to each other in magnitude
(b) Are not equal to each other in magnitude
(c) Cannot be predicted
(d) Are equal to each other

69) Two forces of 12 N and 8 N act upon a body. The resultant force on the body has maximum value of_______?
(a) 4 N (b) 0 N
(c) 20 N (d) 8 N

70) Two equal forces (P each) act at a point inclined to each other at an angle of 120°. The magnitude of their resultant is
(a) P/2 (b) P/4
(c) P (d) 2P

71) Two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A}+\vec{B}=\vec{A}-\vec{B}$, then___________?
(a) $\vec{A}.\vec{B}=0$ (b) $\vec{A}\times\vec{B}=0$
(c) $\vec{A}=0$ (d) $\vec{B}=0$

72) If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ perpendicular to the vector $4\hat{j}-4\hat{i}+\alpha\hat{k}$ thenn the value of $\alpha$ will be________?
(a) -1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

73) If two vectors $2\hat{i}+3\hat{j}-\hat{k}$ and $-4\hat{i}-6\hat{j}-\lambda\hat{k}$ are parallel to each other then value of $\lambda$ will be__________?
(a) 0 (b) 2 (c) 3 (d) 4

74) A body, acted upon by a force of 50 N is displaced through a distance 10 meter in a direction making an angle of 60° with the force. The work done by the force be
(a) 200 J (b) 100 J
(c) 300 (d) 250 J

75) A particle moves from position $3\hat{i}+2\hat{j}-6\hat{k}$ to $14\hat{i}+13\hat{j}+9\hat{k}$ due to a uniform force of $4\hat{i}+\hat{j}+3\hat{k}$ If the displacement in meters then work done will be
(a) 100 J (b) 200 J
(c) 300 J (d) 250 J

76) If for two vector $\vec{A}$ and $\vec{B}$ sum $(\vec{A}+\vec{B})$ is perpendicular to the difference $(\vec{A}-\vec{B})$. The ratio of their magnitude is____?
(a) 1 (b) 2 (c) 3 (d) None

77) The angle between the vectors $\vec{A}$ and $\vec{B}$ is $\theta$ The value of the triple product $A.(\vec{B} \times \vec{A})$ is________?
(a) $A^2B$ (b) Zero
(c) $A^2 B\sin\theta$ (d) $A^2 B\cos\theta$

78) If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ then the angle between A and B is________?
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\pi$ (d) $\frac{\pi}{4}$

79) If $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ then the value of $|\vec{A} \times \vec{B}|$ will be________?
(a) $8\sqrt{2}$ (b) $8\sqrt{3}$ (c) $8\sqrt{5}$ (d) $5\sqrt{8}$

80) If $\vec{A} \times \vec{B} = \vec{C}$, then which of the following statements is wrong________?
(a) $\vec{C} \perp \vec{A}$ (b) $\vec{C} \perp \vec{B}$
(c) $\vec{C} \perp (\vec{A} + \vec{B})$ (d) $\vec{C} \perp (\vec{A} \times \vec{B})$

81) Consider two vectors $\vec{F_1} = 2\hat{i} + 5\hat{j}$ and $\vec{F_2} = 3\hat{j} + 4\hat{k}$ The magnitude of the scalar product of these vectors is________?
(a) 20 (b) 23 (c) $5\sqrt{33}$ (d) 26

82) Consider a vector $\vec{F} = 4\hat{i} - 3\hat{j}$ Another vector that is perpendicular to $\vec{F}$ is________?
(a) $4\hat{i} + 3\hat{j}$ (b) $6\hat{i}$
(c) $7\hat{k}$ (d) $3\hat{i} - 4\hat{j}$

83) Two vectors $\vec{A}$ and $\vec{B}$ are at right angles to each other, when________?
(a) $\vec{A} + \vec{B} = 0$ (b) $\vec{A} - \vec{B} = 0$
(c) $\vec{A} \times \vec{B} = 0$ (d) $\vec{A}.\vec{B} = 0$

84) If $|\vec{V_1} + \vec{V_2}| = |\vec{V_1} - \vec{V_2}|$ and $V_2$ is finite then________?
(a) $V_1$ is paarallel to $V_2$ (b) $\vec{V_1} = \vec{V_2}$
(c) $V_1$ and $V_2$ are perpendiicular
(d) $|\vec{V_1} = \vec{V_2}|$

85) The angle between two vectors $-2\hat{i} + 3\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$ is________?
(a) 0° (b) 90° (c) 180° (d) None

86) The angle between the vectors $(\hat{i} + \hat{j})$ and $(\hat{j} + \hat{k})$ is________
(a) 30° (b) 45° (c) 60° (d) 90°

87) If $\vec{P}.\vec{Q} = PQ$, then angle between $\vec{P}$ and $\vec{Q}$ is________
(a) 0° (b) 30° (c) 45° (d) 60°

88) The angle between the two vectors $A = 5\hat{i} + 5\hat{j}$ and $B = 5\hat{i} - 5\hat{j}$ will be________
(a) Zero (b) 45° (c) 90° (d) 180°

89) The vector $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other. The positive value of a is________?
(a) 3 (b) 4 (c) 9 (d) 13

90) A particle moves in the x-y plane under the action of a force $\vec{F}$ such that the value of its linear momentum $(\vec{P})$ at anytime t is $P_x = 2\cos t, P_y = 2\sin t$. The angle $\theta$ between $\vec{F}$ and $\vec{P}$ at a given time t will be________?
(a) $\theta = 0°$ (b) $\theta = 30°$
(c) $\theta = 90°$ (d) $\theta = 180°$

91) The area of the parallelogram represented by the vectors $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + 4\hat{j}$ is________?
(a) 14 units (b) 7.5 units
(c) 10 units (d) 5 units

92) A vector $\vec{F_1}$ is along the positive X-axis If its vector product with another vector $\vec{F_2}$ is zero then $\vec{F_2}$ could be________?
(a) $4\hat{j}$ (b) $(-\hat{i} + \hat{j})$ (c) $(\hat{j} + \hat{k})$ (d) $(-4\hat{i})$

93) If for two vectors $\vec{A}$ and $\vec{B}$, $\vec{A} \times \vec{B} = 0$, the vectors________?
(a) Are perpendicular to each other
(b) Are parallel to each other
(c) Act at an angle of 60°
(d) Act at an angle of 30°

94) The angle between vectors $(\vec{A} \times \vec{B})$ and $(\vec{B} \times \vec{A})$________?
(a) zero (b) $\pi$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

95) What is the angle between $(\vec{P} + \vec{Q})$ and $(\vec{P} \times \vec{Q})$________?
(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\pi$

96) The resultant of the two vectors having magnitude 2 and 3 is 1 What is their cross product
(a) 6 (b) 3 (c) 1 (d) 0

97) Let $\vec{A} = \hat{i}A\cos\theta + \hat{j}A\sin\theta$ be any vector Another vector $\vec{B}$ which is normal to A is________?
(a) $\hat{i}B\cos\theta + \hat{j}B\sin\theta$ (b) $\hat{i}B\sin\theta + \hat{j}B\cos\theta$
(c) $\hat{i}B\sin\theta - \hat{j}B\cos\theta$ (d) $\hat{i}B\cos\theta + \hat{j}B\sin\theta$

98) The angle between two vectors given by $6\hat{i} + 6\hat{j} - 3\hat{k}$ and $7\hat{i} + 4\hat{j} + 4\hat{k}$ is________?
(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}$([illegible])

(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{\sqrt{5}}{3}\right)$

99) A vector $\vec{A}$ points vertically upward and $\vec{B}$ points towards north. The vector product $\vec{A}\times\vec{B}$ is__________?
(a) Zero (b) Along west
(c) Along east (d) Vertically downward

100) Angle between the vectors $(\hat{\imath}+\hat{\jmath})$ and $(\hat{\jmath}-\hat{k})$ ________?
(a) 90° (b) 0° (c) 180° (d) 60°

101) if $|\vec{A}\times\vec{B}| = |\vec{A}.\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be__________?
(a) 30° (b) 45° (c) 60° (d) 90°

102) three vectors $\vec{a}$ $\vec{b}$ and $\vec{c}$ satisfy the relation $\vec{a}.\vec{b}=0$ and $\vec{a}.\vec{c}=0$ the vector $\vec{a}$ is parallel to________?
(a) $\vec{b}$ (b) $\vec{c}$ (c) $\vec{b}.\vec{c}$ (d) $\vec{b}\times\vec{c}$

103) The diagonals of a parallelogram are $2\hat{\imath}$ and $2\hat{\jmath}$. What is the area of the parallelogram
(a) 0.5 units (b) 1 unit
(c) 2 units (d) 4 units

104) Two vector A and B have equal magnitudes. Then the vector A + B is perpendicular to
(a) A x B (b) A – B
(c) 3A – 3B (d) All of these

105) The value of $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$ is________?
(a) 0 (b) $A^2-B^2$
(c) $\vec{B}\times\vec{A}$ (d) $2(\vec{B}\times\vec{A})$

106) if $\vec{A}$ and $\vec{B}$ are perpendicular vectors and vector $\vec{A}=5\hat{\imath}+7\hat{\jmath}+3\hat{k}$ and $\vec{B}=2\hat{\imath}+2\hat{\jmath}-a\hat{k}$. The value of a is__________?
(a) – 2 (b) 8
(c) – 7 (d) – 8

107) A force vector applied on a mass is represented as $\vec{F}=6\hat{\imath}-8\hat{\jmath}+10\hat{k}$ and accelerates with 1 m/s². What will be the mass of the body in kg_________?
(a) $10\sqrt{2}$ (b) 20 (c) $2\sqrt{10}$ (d) 10

108) What is the value of linear velocity, if $\vec{\omega}=3\hat{\imath}-4\hat{\jmath}+\hat{k}$ and $\vec{r}=5\hat{\imath}-6\hat{\jmath}+6\hat{k}$________?
(a) $6\hat{\imath}-2\hat{\jmath}+3\hat{k}$ (b) $6\hat{\imath}-2\hat{\jmath}+8\hat{k}$
(c) $4\hat{\imath}-13\hat{\jmath}+6\hat{k}$ (d) $-18\hat{\imath}-13\hat{\jmath}+2\hat{k}$

109) Dot product of two mutual perpendicular vector is
(a) 0 (b) 1 (c) ∞ (d) None

110) When $\vec{A}.\vec{B}=-|A||B|$ then?
(a) $\vec{A}$ and $\vec{B}$ are perpendicular to each other
(b) $\vec{A}$ and $\vec{B}$ act in the same direction
(c) $\vec{A}$ and $\vec{B}$ act in the opposite direction
(d) $\vec{A}$ and $\vec{B}$ can act in any direction

111) If a vector $\vec{A}$ is parallel to another vector $\vec{B}$ then the resultant of the vector $\vec{A}\times\vec{B}$ will be equal to_______?
(a) A (b) $\vec{A}$ (c) Zero vector (d) Zero

112) If the resultant of n forces of different magnitudes acting at a point is zero, then the minimum value of n is?
(a) 1 (b) 2 (c) 3 (d) 4

113) Can the resultant of 2 vectors be zero_________?
(a) Yes, when the 2 vectors are same in magnitude and direction (b) No
(c) Yes, when the 2 vectors are same in magnitude but opposite in sense
(d) Yes, when the 2 vectors are same in magnitude making angle of $\frac{2\pi}{3}$ with each other

114) The sum of the magnitudes of two forces acting at point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the magnitudes of forces_______?
(a) 12, 5 (b) 14, 4
(c) 5, 13 (d) 10, 8

115) The angle between two vectors of magnitude 12 and 18 units when their resultant is 24 units is
a) 63°51' b) 75°52'
c) 82°31' d) 89°16'

116) If a unit vector is represented by 0.5i + 0.8j + ck, the value of c is ..
a) 1 b) √(0.11) c) √(0.01) d) 0.39

117) The vector A and B are such that |A + B| = | A - B | The angle between the two vectors is ..
a) 60° b) 75° c) 45° d) 90°

118) The magnitude of vectors A, B, C are 3, 4 and 5 units respectively. If A+ B=C then the angle between A and B is ..
a) π/2 b) cos-1 0.6 c) tan-1 7/5 d) π/4

119) If $|A \times B| = AB\sqrt{3}$, then the value of $|A+B|$ is ..
a) $(A^2 + B^2 + AB\sqrt{3})^{1/2}$
b) $(A^2 + B^2 + AB)^{1/2}$
c) $(A^2 + B^2 + AB/\sqrt{3})^{1/2}$
d) A + B

120) If the angle between the vectors A and B is θ, the value of the product (B ×A) . A is equal to
a) $BA^2\sin\theta$ b) $BA^2\cos\theta$
c) $BA^2\sin\theta\cos\theta$ d) zero

121) The result of (B × 0) will be equal to
a) zero b) B c) zero vector d) unit vector

122) A particle moving with a velocity v=5i -4j+3k m/s under the influence of a constant force F=20i + 15j - 5k
The instantaneous power applied to the particle is ...
a) 45 J/s b) 35 J/s c) 25 J/s d) 195 J/s

123) If A ×B=B×A, then angle between A and B is ..
a) π/2 b) π/3 c) π d) π/4

124) A particle has an initial velocity 3i + 4j and an acceleration of 0.4i + 0.3j. Its speed after 10s is
a) $7\sqrt{2}$ units b) 7-unit c) 8.5 units d) 10 unit

125) The vector P=ai+aj+3k and Q=ai -2j-k are perpendicular to each other. The positive value of 'a' is
a) 23 b) 9 c) 8 d) 3

126) Two vectors A and B are such that A + B=C and $A^2 + B^2 = C^2$
If θ is the angle between positive direction of A and B then the correct statement is ..
a) θ=π b) θ=2π/3 c) θ=0 d) θ=π/2

127) If two numerically equal force P and Q acting at a point produces a resultant force of magnitude P then the angle between the two original forces is
a) 120° b) 90° c) 0° d) 45°

128) What is the dot product of two vectors of magnitude 3 and 5, if angle between them is 60°?
a) 5.2 b) 7.5 c) 8.4 d) 8.6

129) Consider a F=4i - 3j. Another vector perpendicular to F is
a) 4i + 3j b) 6i c) 7k d) 3i - 4j

130) If A=B + C and the magnitude of A, B, C are 5, 4, 3 units, the angle between A and C is
a) cos-1(3/5) b) cos-1(4/5)
c) π/2 d) sin-1(3/5)

131) Out of the following pairs, the resultant of which cannot be zero
a) 10, 10, 10 b) 10, 10, 20
c) 10,20, 20 d) 10, 20, 40

132) A particle is acted up on by two forces of 3N and 4N simultaneously Which of the following is most correct
a) The resultant of these forces is 7N
b) The resultant of these forces is 1N
c) The resultant of these forces is 4N
d) The resultant of these forces lie between 1 N and 7 N

133) The vector sum of N coplanar forces each of magnitude F, when each force is making an angle (2π / N) with the preceding one is
a) NF b) NF /2 c) 3F/2 d) zero

134) If $A = A\cos\theta i - A\sin\theta j$ be any vector. Another vector B, which is normal to A can be expressed as
a) $B\cos\theta i - B\sin\theta j$ b) $B\cos\theta i + B\sin\theta j$
c) $B\sin\theta i - B\cos\theta j$ d) $B\sin\theta i + B\cos\theta j$

135) Out of following pairs, the resultant of which cannot be 4 Newton
a) 2N and 2N b) 2N and 6 N
c) 2N and 4N d) 4N and 3N

136) Let F be the force acting on a particle having position vector r and The the torque of this force about the origin . Then...
a) $r \cdot T = 0$ and $F \cdot T \neq 0$
b) $r \cdot T \neq 0$ and $F \cdot T = 0$
c) $r \cdot T \neq 0$ and $F \cdot T \neq 0$
d) $r \cdot T = 0$ and $F \cdot T = 0$

137) Two like parallel forces P and 3P are 40cm apart. If the directions of P is reversed, then their resultant shifts through a distance
a) 30 cm b) 40 cm c) 50 cm d) 60 cm

138) The simple sum of two forces acting at a point is 16N and there sum is 8N and its direction is perpendicular to the smaller force, then the forces are..
a) 6 newton and 10 newton
b) 8 newton and 8 newton
c) 4 newton and 12 newton
d) 2 newton and 14 newton

139) If A=B + C and the magnitude of A, B and C are 5, 4 and 3 units respectively. The angle between A and C is ...

a) cos-1(3/5) b) cos-1(4/5) c) π/2 d) sin-1(3/5)

140) If A=$iA\cos\theta - jA\sin\theta$ be any vector. Another vector B, which is normal to B can be expressed as
a) $B\cos\theta i + B\sin\theta j$ b) $B\cos\theta i - B\sin\theta j$
c) $B\sin\theta i + B\cos\theta j$ d) $B\sin\theta i - B\cos\theta j$

141) Given that A + B=C and that C is ⊥ to A. Further if |A|=|C| , then what is the angle between A and B
a) $\pi/4\ radian$ b) $\pi/2\ radian$
c) $3\pi/4\ radian$ d) $\pi\ radian$

142) If two non-zero vectors A and B obey the relation A+B=-B , the angle between them is
a) 120° b) 90° c) 60° d) 0°

143) Two like parallel forces of 4 N and 6N acts on a body at a separation of 4m. The resultant will be
a) 2.4 m from 4 N b) 2.4m from 6N
c) 2.0 m from 6N d) 1. m from 4N

144) Choose the wrong statement
a) Three vectors of different magnitudes may be combined to give zero resultant
b) Two vectors of different magnitudes can be combined to give a zero resultant
c) The product of a scalar and vector is a vector quantity
d) All above statements are wrong statement

145) Two forces, one of 10N and other 6N act upon a body. The direction of the forces are unknown, the resultant force on the body is
a) Not less than 4N and not more than 16N
b) More than 15N always
c) More than 6N d) More than 10N

146) A vector A of magnitude 10 units and another vector of magnitude 6 units are acting at 60° to each other. The magnitude of vector product of two vector is
a) 30 units b) 60 unit
c) 30√3 unit d) none of the above

147) The magnitude of vector cannot be
a) unity b) positive c) negative d) zero

148) The area of parallelogram with sides as A=2i + 3j and B=i + 4j
a) 5 units b) 10 units c) 15 units d) 20

149) If A.B=AB then the angle between A and B is
a) 0° b) 45°
c) 90° d) 180°

150) Given A + B=R and A +2B is perpendicular to A, then
a) 2=R b) B=2R
c) B=R d) B2=2R2

151) Projection of vector A over B is
a) $\vec{A}.\vec{B}$ b) $\vec{A}.\hat{B}$
c) $\hat{B}.\vec{A}$ d) $\hat{A}.\hat{B}$

152) For any two vectors A and B if A.B=|A×B|, the magnitude of C=A + B is equal to
a) $\sqrt{(A^2 + B^2)}$
b) A + B
c) $\sqrt{A^2 + B^2 + \left(\frac{AB}{\sqrt{2}}\right)}$
d) $\sqrt{A^2 + B^2 + \sqrt{2} \times AB}$

153) A vector x, when added to two vectors A=3i -5j + 7k and B=2i + 4j - 3k gives a unit vector along y axis. The vector x is
a) -5i + 2j - 4k b) 5i + 2j - 4k
c) -5i - 2j + 4k d) 5i - 2j - 4k

154) At what angle the two forces A+B and A-B acts so that their resultant is √(3A2 + B2)
a) π/6 b) π/3 c) π/2 d) π

155) Resultant of two forces acting at right angles to each other is 1414 dyne. The magnitude of each force is
a) 500 N b) 100 N c) 1000 N d) none

156) The component of 9i +17k along z-axis has magnitude
a) zero b) 17 c) 9 d) 26

157) Which of the following statement is true
a) Energy, force and weight are vector quantities
b) Relative velocity, work and acceleration are vector quantities
c) displacement, force and electric field are vector quantities
d) Velocity, length and acceleration are vector quantities?

158) i×(j×k) is
a) i + j + k b) i - j + k
c) zero vector d) unit vector

159) If A=2i + 3j and B=i + 4j + k , then unit vector along (A+B) is

a) $\frac{3\hat{i}+7\hat{j}+\hat{k}}{\sqrt{59}}$ b) $\frac{2\hat{i}+3\hat{j}}{\sqrt{59}}$

c) $\frac{\hat{i}+4\hat{j}+\hat{k}}{\sqrt{18}}$ d) $\frac{2\hat{i}+3\hat{j}}{\sqrt{13}}$

## ANSWERS WITH EXPLANATION

1) This is a recall question if the angle between two concurrent forces or vector increases then magnitude of their resultant decreases.

2) The final displacement of the girl is the vector from her starting point to her ending point. if you follow the displacement in order you will see that 4m west of her starting point.

3) Magnitude of couple forces is equal to (Force x distance between them) for two forces acting on a same point there is zero distance between them therefore option b is correct.

4) (d) As the multiple of $\hat{j}$ in the given vector is zero therefore this vector lies in XZ plane and projection of this vector on y-axis is zero.

5) (b) If a point have coordinate (x, y, z) then its position vector $x\hat{i}+y\hat{j}+z\hat{k}$ so $3\hat{i}+2\hat{j}+5\hat{k}$.

6) (c) Displacement vector

$$=\Delta x\hat{i}+\Delta y\hat{j}+\Delta z\hat{k}$$
$$=(3-2)\hat{i}+(4-3)\hat{j}+(5-5)\hat{k}=\hat{i}+\hat{j}$$

7) (d) → The component of force in vertical direction

$$=F\cos\theta=F\cos 60^\circ=5\times\frac{1}{2}=2.5N$$

8) (d)

$$|B|=\sqrt{7^2+(24)^2}=\sqrt{625}=25$$

Unit vector in the direction of A will be $\hat{A}=\frac{3\hat{i}+4\hat{j}}{5}$

So, required vector

$$=25\left(\frac{3\hat{i}+4\hat{j}}{5}\right)=15\hat{i}+20\hat{j}$$

9) (a) Let the components of $\vec{A}$ makes angles α, β and γ with x, y and z axis respectively then α = β = γ

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$
$$\Rightarrow 3\cos^2\alpha=1\Rightarrow\cos\alpha=\frac{1}{\sqrt{3}}$$
$$\Rightarrow A_x=A_y=A_z=A\cos\alpha=\frac{A}{\sqrt{3}}$$

10) (b) Unit vector along y axis = $\hat{j}$ so the required vector

$$=\hat{j}-[(\hat{i}-3\hat{j}+2\hat{k})+(3\hat{i}+6\hat{j}-7\hat{k})]$$
$$=-4\hat{i}-2\hat{j}+5\hat{k}$$

11) (b) $\vec{F_3}=\vec{F_1}+\vec{F_2}$

⇨ There should be minimum three coplanar vectors having different magnitude which should be added to give zero resultant.

12) (d) Diagonal of the hall $=\sqrt{l^2+b^2+h^2}$

$$=\sqrt{10^2+12^2+14^2}$$
$$=\sqrt{100+144+190}\Rightarrow\sqrt{400}=20m$$

13) (d)

$$Total\ angle=100\times\frac{\pi}{50}=2\pi$$

So, all the force will pass through one point and all forces will be balanced. i.e their resultant will be zero

14) (d)

$$\vec{r}=\vec{r_2}-\vec{r_1}=(-2\hat{i}-2\hat{j}+0\hat{k})-(4\hat{i}-4\hat{j}+0\hat{k})$$
$$\Rightarrow\vec{r}=-6\hat{i}+2\hat{j}+0\hat{k}$$
$$\Rightarrow|\vec{r}|=\sqrt{(-6)^2+(2)^2+0^2}=\sqrt{36+4}$$
$$=\sqrt{40}=2\sqrt{10}$$

15) (a)

$$\vec{p}=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$$
$$\Rightarrow|\vec{p}|=\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}$$

= 1 *So it is a unit vector.*

16) b is correct

17) (c)

$$\hat{R}=\frac{\vec{R}}{|R|}=\frac{\hat{i}+\hat{j}}{\sqrt{1^2+1^2}}=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$$

18) (c) $\vec{R}=3\hat{i}+\hat{j}+2\hat{k}$

Length in xy plane $=\sqrt{Rx^2+Ry^2}=\sqrt{3^2+1^2}=\sqrt{10}$

19) (a) If the angle between all forces which are equal and lying in one plane are equal then resultant force will be zero.

20) (b)

$$\vec{A}=\hat{i}+\hat{j}\Rightarrow|A|=\sqrt{1^2+1^2}=\sqrt{2}$$
$$\cos\alpha=\frac{A_x}{|A|}=\frac{1}{\sqrt{2}}=\cos 45^\circ$$

21) (c)

22) (d) $\vec{P}+\vec{Q}=P\hat{P}+Q\hat{Q}$

23) (b)

$$\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = -a\omega\sin\omega t\hat{j} + a\omega\cos\omega t\hat{j}$$

As $\vec{r}\cdot\vec{v} = 0$ therefore velocity of the particle is perpendicular to the position vector.

24) Given that vector C is resultant of vector A and B
According to fomula for vector addition
$C^2=A^2 + B^2 + 2AB\cos\theta$
Comparing $C^2=A^2 + B^2$ with above equation
$\cos\theta=0$ since magnitude of A and B is not zero
$\Rightarrow\theta=\pi/2$
Answer: (d)

25) Let θ be the angle made by the particle with x-axis as shown in figure
$\tan\theta$=y co-ordinates / x - coordinate
$\tan\theta=3/\sqrt{3}=\sqrt{3}$
thus $\theta=60^\circ$
Answer: (b)

26) Given two vectors are perpendicular to each other therefore dot product of vector A and Vector B will be zero

$$\vec{A}.\vec{B} = (2\hat{i} + 3\hat{j} + 8\hat{k}).(-4\hat{i} + 4\hat{j} + a\hat{k})$$

$-8+12+8a=0$
$a=-1/2$
Answer:(b)

27) Let vector P be the vector sum of vector A and Vector B
Let Vector Q be the vector difference of vector A and Vector B
Since vector P and Q are perpendicular there dot product will be zero

$$(\vec{A}+\vec{B}).(\vec{A}-\vec{B}) = 0$$

$$A^2 - B^2 = 0$$

$$|\vec{A}| = |\vec{B}|$$

Answer: (c)

28) (b) Magnitude of unit vector = 1

$$\Rightarrow \sqrt{(0\cdot5)^2 + (0\cdot8)^2 + C^2} = 1$$

By solving we get $c = \sqrt{0.11}$

29) (b)

$$Displacement = \overline{AC} = \overline{AB} + \overline{BC}$$

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{(4000)^2 + (300)^2}$$

$$= 500m$$

$$Distance = AB + BC = 400 + 300$$

$$= 700\ m$$

30) (a) Resultant of vectors $\vec{A}$ and $\vec{B}$

$$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$$

$$\vec{R} = 3\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{3^2+6^2+(-2)^2}} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$$

31) (a)

$\phi = \vec{B}\cdot\vec{A}$ *this equation* $\vec{A}$ *is area vector*

32) (d),

$$\Rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A||B|}$$

$$= \frac{9+16+25}{\sqrt{9+16+25}\sqrt{9+16+25}} = \frac{50}{50} = 1$$

$$\Rightarrow \cos\theta = 1 \quad \therefore \theta = \cos^{-1}(1)$$

33) : (a)

$$\Rightarrow \frac{\vec{A}\cdot\vec{B}}{|\hat{i}+\hat{j}|} = \frac{(2\hat{i}+3\hat{j})(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

34)

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A||B|}$$

$$= \frac{9+16-25}{\sqrt{9+16+25}\sqrt{9+16+25}}$$

$$= \frac{9+16-25}{50} = 0$$

$$\Rightarrow \cos\theta = 0, \theta = \cos^{-1}0^\circ \Rightarrow \theta = 90^\circ$$

35) : (a) For 17 N both the vector should be parallel i.e. angle between them should be zero.
For 7 N both the vectors should be antiparallel i.e. angle between them should be 180°
For 13 N both the vectors should be perpendicular to each other i.e. angle between them should be 90°

36) (b)

$$\vec{A}+\vec{B} = 4\hat{i} - 3\hat{j} + 6\hat{i} + 8\hat{j} = 10\hat{i} + 5\hat{j}$$

$$|\vec{A}+\vec{B}| = \sqrt{(10)^2 + (5)^2} = 5\sqrt{5}$$

$$\tan\theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}(\frac{1}{2})$$

37) (b)

$$\Rightarrow \vec{A} - 2\vec{B} + 3\vec{C} = (2\hat{i}+\hat{j}) - 2(3\hat{j}-\hat{k}) + 3((6\hat{i}-2\hat{k})$$

$$\Rightarrow 2\hat{i} + \hat{j} - 6\hat{j} + 2\hat{k} + 18\hat{i} - 6\hat{k}$$

$$= 20\hat{i} - 5\hat{j} - 4\hat{k}$$

38) (b)

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

By substituting, A = F, B = F and R = F we get

$$\cos\theta = \frac{1}{2}, \theta = 120^\circ$$

39) (a)

40) (d) If two vectors $\vec{A}$ and $\vec{B}$ are given then the resultant $R_{max} = A + B = 7N$ and $R_{min} = 4 - 3 = 1N$, i.e. net force on the particle is between 1 N and 7 N.

41) (b) If $\vec{C}$ lies outside the plane then resultant force cannot be zero

42) (d)

43) (c) $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90^\circ} \rightarrow = \sqrt{F_1^2 + F_2^2}$

44) (a)

45) (c)

46) (c) $C = \sqrt{A^2 + B^2}$

The angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{2}$

47) (c) $R = \sqrt{12^2 + 5^2 + 6^2} = \sqrt{144 + 25 + 36} = \sqrt{205} = 14.31\ m$

48) (c) $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}, \vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$

$$|\vec{A}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$$

$$|\vec{B}| = \sqrt{1^2 + (-3)^2 + 5^2} = \sqrt{1 + 9 + 25} = \sqrt{35}$$

$$|\vec{C}| = \sqrt{2^2 + 1^2 + (-4)^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$$

As $B = \sqrt{A^2 + B^2}$ therefore ABC will be right angled triangle

49) (b) $\vec{C} + \vec{A} = \vec{B}$, The value of C lies between A + B and A – B

$$\therefore |\vec{C}| < |\vec{A}| \text{ and } |\vec{C}| < |\vec{B}|$$

50) (d)

51) (d) Here all the three force will not keep the particle in equilibrium so the net force will not be zero and the particle will move with an acceleration.

52) (a) $A + B = 16\ (given)$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \tan 90^\circ \ldots(i)$$

$$\therefore A + B\cos\theta = 0 \rightarrow \cos\theta = \frac{-A}{B} \ldots(ii)$$

$$8 = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

By solving eq. (i), (ii) and (iii) we get A = 6N, B = 10N

53) (c)

$|\vec{P}| = 5, |\vec{Q}| = 12, |\vec{R}| = 13 \rightarrow \cos\theta = \frac{Q}{R} = \frac{12}{13} \therefore \theta = \cos^{-1}\left(\frac{12}{13}\right)$

54) (b)

$$\frac{B}{2} = \sqrt{A^2 + B^2 + 2AB\cos\theta} \ldots(i)$$

$$\therefore \tan 90^\circ = \frac{B\sin\theta}{A + B\cos\theta} \rightarrow A + B\cos\theta = 0 \therefore \cos\theta = -\frac{A}{B}$$

Now from equation(i) $\frac{B^2}{4} = A^2 + B^2 - 2A^2 \rightarrow = \sqrt{3}\frac{B}{2} \rightarrow \cos\theta = -\frac{A}{B} = -\frac{\sqrt{3}}{2} \rightarrow \therefore \theta = 135^\circ$

55) (a) Resultant $\vec{R} = \vec{P} + \vec{Q} + \vec{P} - \vec{Q} = 2\vec{P} \rightarrow$ the angle between $\vec{P}$ and $2\vec{P} = 0^\circ$

56) (b) $\tan 90^\circ = \frac{Q\sin\theta}{P + Q\cos\theta} \rightarrow P + Q\cos\theta = 0$ and $\cos\theta = \frac{-P}{Q} \therefore \theta = \cos^{-1}\left(\frac{-P}{Q}\right)$

57) (a) According to problem $P + Q = 3$ and $P - Q = 1$

By solving we get $P = 2$ and $Q = 1 \therefore \frac{P}{Q} = 2 \rightarrow P = 2Q$

58) (c)

59) (c)

60) (c)

61) (d) is correct $F_1 + F_2 + F_3 = 0 \rightarrow 4\hat{i} + 6\hat{j} + F_3 = 0 \therefore \vec{F_3} = -4\hat{i} + 6\hat{j}$

62) (d) is correct Resultant velocity $= \sqrt{20^2 + 15^2} = \sqrt{400 + 225} = \sqrt{625} = 25\ km/hr$

63) (a) is correct

$$C = \sqrt{A^2 + B^2} \rightarrow \sqrt{3^2 + 4^2} = 5$$

$\rightarrow$ Angle between $\vec{A}$ and $\vec{B}$ is $\pi/2$

64) (b) $\cos\theta = \frac{\vec{F_1}.\vec{F_2}}{|\vec{F_1}||\vec{F_2}|}$

$$\rightarrow \frac{(5\hat{i} + 10\hat{j} - 20\hat{k}).(10\hat{i} - 5\hat{j} - 15\hat{k})}{\sqrt{25 + 100 + 400}\sqrt{100 + 25 + 225}} = \frac{50 - 50 + 300}{\sqrt{525}\sqrt{350}} \rightarrow \cos\theta = \frac{1}{\sqrt{2}} \therefore \theta = 45^\circ$$

65) (d) If two vectors A and B are given then Range of their resultant can be written as $(A - B) \le R \le (A + B)$ i.e. $R_{max} = A + B$ and $R_{min} = A - B$

If B = 1 and A = 4 then their resultant will lie in between 3N and 5N. It can never be 2N

66) (a) In N forces of equal magnitude works on a single point and their resultant is zero then angle between any two forces is given $\theta = \frac{360}{N} = \frac{360}{3} = 120^\circ$

If these three vectors are represented by three sides of triangle then they form ecuilateral triangle

**67)** (c) Resultant of two vectors $\vec{A}$ and $\vec{B}$ can be given by

$R = \vec{A} + \vec{B} \rightarrow |\vec{R}| = |\vec{A} + \vec{B}|$
$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$
$if\ \theta = 0, then |\vec{R}| = A + B = |\vec{A}| + |\vec{B}|$

**68)** (a) If two vectors are perpendicular then their dot product must be equal to zero. According to problem

$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$
$\rightarrow \vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B} = 0$
$\rightarrow A^2 - B^2 = 0 \rightarrow A^2 = B^2$
$\therefore A = B, i.e.$ *two vectors are equal to each other in magnitude*

**69)** $R_{max} = A + B \rightarrow when\ \theta = 0° \therefore R_{max} = 12 + 8 = 20N$

**70)** (c) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ $if\ A = B = P\ and\ \theta = 120°\ then\ R = P$

**71)** (d)

**72)** (c) Given vectors can be rewritten as $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$ and $\vec{B} = 4\hat{\jmath} - 4\hat{\imath} + a\hat{k}$ Dot product of these vectors should be equal to zero because they are perpendicular.

$\therefore \vec{A}.\vec{B} = -8 + 12 + 8a = 0 \rightarrow 8a = -4$
$\rightarrow a = -1/2$

**73)** (b) $Let\ \vec{A} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}\ and\ \vec{B} = -4\hat{\imath} - 6\hat{\jmath} - \lambda\hat{k}$

$\vec{A}$ *and* $\vec{B}$ *are paralleII to each other*

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}\ i.e... \frac{2}{-4} = \frac{3}{-6} = -\frac{1}{\lambda} \rightarrow \lambda = 2$

**74)** (d) $W = \vec{F}.\vec{S} = FS\cos\theta = 50 \times 10 \times \cos 60° = 50 \times 10 \times \frac{1}{2} = 250\ J$

**75)** (a) $S = \vec{r_2} - \vec{r_1} \rightarrow W = \vec{F}.\vec{S} = (4\hat{\imath} + \hat{\jmath} + 3\hat{k}).(11\hat{\imath} + 11\hat{\jmath} + 15\hat{k})$
$= (4 \times 11 + 1 \times 11 + 3 \times 15) = 100J$

**76)** (a) $(\vec{A} + \vec{B})$ *is perpendicular to* $(\vec{A} - \vec{B})$. *thus* $(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$

$or\ A^2 + \vec{B}.\vec{A} - \vec{A}.\vec{B} - B^2 = 0$

Because of commutative property of dot product $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

$\therefore A^2 - B^2 = 0\ or\ A$
$= B,$ *thus the ratio of magnitude* $\frac{A}{B} = 1$

**77)** (b)

$A.(\vec{B} \times \vec{A}) = \vec{A}.\vec{C}$, *and Here* $\vec{C} = \vec{B} \times \vec{A}$
$\rightarrow$ *Which is perpendicular to both vector* $\vec{A}$ *and* $\vec{B} \therefore \vec{A}.\vec{C} = 0$

**78)** (c) We know that $\vec{A} \times \vec{B} = (-\vec{B} \times \vec{A})$ because the angle between these two is always 90°. But if the angle between $\vec{A}$ and $\vec{B}$ is 0 or $\pi$. Then $\vec{A} \times \vec{B} = \vec{B} \times \vec{A} = 0$

**79)** (b) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$= (1 \times 4 - 2 \times -2)\hat{\imath} + (2 \times 2 - 4 \times 3)\hat{\jmath} + (3 \times -2 - 1 \times 2)\hat{k} \rightarrow$
$= 8\hat{\imath} - 8\hat{\jmath} - 8\hat{k}$

$\therefore$ *Magnitude of* $\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}|$
$= \sqrt{8^2 + (-8)^2 + (-8)^2}$
$= 8\sqrt{3}$

**80)** (d) From the property of vector product, we notice that $\vec{C}$ must be Perpendicular to the plane formed by vector $\vec{A}$ and $\vec{B}$ Thus $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ and $(\vec{A} + \vec{B})$ also, must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ must be perpendicular to $(\vec{A} + \vec{B})$ also but the cross product $(\vec{A} \times \vec{B})$ gives a vector $\vec{C}$ which can not be perpendicular to itself. Thus the last statement is wrong.

**81)** (d) $\vec{F_1}.\vec{F_2} = (2\hat{\imath} + 5).(3\hat{\jmath} + 4\hat{k}) = 6 + 20 = 20 + 6 = 26$

**82)** (c) Force F lie in the x-y plane so a vector along z-axis will be perpendicular to F

**83)** (d)

$\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|.\cos\theta = \vec{A}.\vec{B}.\cos 90° = 0$

**84)** (c) *According to problem*

$|\vec{V_1} + \vec{V_2}| = |\vec{V_1} - \vec{V_2}|$

$|\vec{V_{net}}| = |\vec{V_{net}}'|$

$\rightarrow$ *So* $V_1$ *and* $V_2$ *are mutually perpendicular*

**85)** (b)

$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{-2 + 6 - 4}{\sqrt{14}\sqrt{21}} = 0 \therefore \theta = 90°$

**86)** (c) $(\hat{\imath} + \hat{\jmath}).(\hat{\jmath} + \hat{k}) = 0 + 0 + 1 + 0 = 1$

$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{1}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2} \therefore \theta = 60°$

87) (a) $cos\theta = \frac{\vec{P}.\vec{Q}}{PQ} = 1 \therefore$
$\theta = 60°$

88) (c) $\vec{A}.\vec{B} = 0 \therefore \theta = 90°$

89) (a) $\vec{P}.\vec{Q} = 0 \therefore a^2 - 2a - 3 = 0 \rightarrow a = 3$

90) (c) $P_x = 2cost, P_y = 2sint \therefore \vec{P} = 2cost\hat{\imath} + 2sint\hat{\jmath}$

$$\vec{F} = \frac{d\vec{P}}{dt} = -2\sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath} \rightarrow \vec{F}.\vec{P} = 0$$
$$\therefore \theta = 90°$$

91) (d) $|\vec{A} \times \vec{B}| = |(2\hat{\imath} + 3\hat{\jmath}) \times (\hat{\imath} + 4\hat{\jmath})| = |5\hat{k}| = 5\ units$

92) (d)

93) (b)
$\vec{A} \times \vec{B} = 0 \therefore sin\theta = 0 \therefore \theta = 0°$
Two vectors will be parallel to each other.

94) $(\vec{A} \times \vec{B})$ and $(\vec{B} \times \vec{A})$ are parallel and opposite to each other. So the angle will be $\pi$.

95) (b) Vector $(\vec{P} + \vec{Q})$ lies in a plane and vector $(\vec{P} \times \vec{Q})$ is perpendicular to this plane i.e. the angle between given vectors is $\frac{\pi}{2}$

96) (d)
$\sqrt{2^2 + 3^2 + 2 \times 2 \times 3 \times cos\theta} = 1$, By solving we get $\theta = 180° \therefore \vec{A} \times \vec{B} = 0$

97) (c) Dot product of two perpendicular vector will be zero.

98) (d) is correct
$$cos\theta = \frac{\vec{A}.\vec{B}}{AB}$$
$$= \frac{24 + 24 - 12}{\sqrt{36 + 36 + 9}\sqrt{49 - 16 + 16}} = \frac{56}{9\sqrt{71}}$$
$$cos\theta = \frac{56}{9\sqrt{71}} \therefore sin\theta = \frac{\sqrt{5}}{3}\ or\ 6$$
$$= \sin^{-1}\left(\frac{\sqrt{5}}{3}\right)$$

99) (b) Direction of vector A is along z-axis
$\therefore \vec{A} = a\hat{k}$
Direction of vector B is towards north
$\vec{B} = b\hat{\jmath}$
$$Now. \vec{A} \times \vec{B} = a\hat{k} \times b\hat{\jmath} = ab(-\hat{\jmath})$$
$\therefore$ The direction is $\vec{A} \times \vec{B}$ is along west

100) (d)
$$cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2} = \therefore \theta = 60°$$

101) (b) $|\vec{A} \times \vec{B}| = |\vec{A}.\vec{B}| \rightarrow ABsin\theta = ABcos\theta \rightarrow tan\theta = 1 \quad \therefore \theta = 45°$

102) (d)
$\vec{a}.\vec{b} = 0$, i.e., $\vec{a}$ and $\vec{b}$ will be perpendicular to each other
$\vec{a}.\vec{c} = 0$, i.e ... $\vec{a}$ and $\vec{c}$ will be perpendicular to each other
$\vec{b} \times \vec{c}$ will be a vector perpendicular to both $\vec{b}$ and $\vec{c}$
So $\vec{a}$ is parallel to $\vec{b} \times \vec{c}$

103) (d) $Area = |2\hat{\imath} \times 2\hat{\jmath}| = |4\hat{k}| = 4\ units$

104) (a)
$\vec{A} \times \vec{B}$ is a vector perpendicular to plane $\vec{A} + \vec{B}$ and hence perpendicular to $\vec{A} + \vec{B}$

105) (d) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B}) = \vec{A} \times \vec{A} - \vec{A} \times \vec{B} + \vec{B} \times \vec{A} - \vec{B} \times \vec{B} = 0 - \vec{A} \times \vec{B} + \vec{B} \times \vec{A} = 2(\vec{B} \times \vec{A})$

106) (d) For perpendicular vector $\vec{A}.\vec{B} = 0$
$$\Rightarrow (5\hat{\imath} + 7\hat{\jmath} + 3\hat{k}).(2\hat{\imath} + 2\hat{\jmath} - a\hat{k}) = 0$$
$$\Rightarrow 10 + 14 + 3a = 0 \Rightarrow a = -8$$

107) (a) $Mass = \frac{|\vec{F}|}{a} \Rightarrow \frac{\sqrt{36+64+100}}{1} = 10\sqrt{2}\ kg$

108) (d) $\vec{v} = \vec{\omega} \times \vec{r} = \begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{matrix} \Rightarrow$
$-18\hat{\imath} - 13\hat{\jmath} + 2\hat{k}$

109) (a)

110) (c) $\vec{A}.\vec{B} = AB\cos\theta \Rightarrow$
In the problem $\vec{A}.\vec{B} = -|A||B|$
i.e.. $cos\theta = -1, \therefore \theta = 180°$
i.e.. $\vec{A}$ and $\vec{B}$ act in the opposite direction

111) (c) $\vec{A} \times \vec{B} = AB\ sin\theta\hat{n}$, for parallel vectors $\theta = 0°\ or\ 180°, \sin\theta = 0$
$$\therefore \vec{A} \times \vec{B} = \vec{0}$$

112) (c) If vectors are of equal magnitude then two vectors can give zero resultant, if they work in opposite direction. But if the vectors are of different magnitudes then minimum three vectors are required to give zero resultant.

113) (c)

114) (c) Let P be the smaller force and Q be the greater force then according to problem $-P+Q=18$ ...(i)

$R=\sqrt{P^2+Q^2+2PQ\cos\theta}$
$=12$ ...(ii)

$\tan\phi=\dfrac{Q\sin\theta}{P+Q\cos\theta}=\tan 90=\infty$

$\therefore P+Q\cos\theta$
$=0$ ...(iii)

By solving (i), (ii) and (iii) we will get $P=5$, and $Q=13$

115) We know that resultant vector

$R^2=A^2+B^2+2AB\cos\theta$

$24^2=12^2+18^2+2(18)(12)\cos\theta$

$\cos\theta=108/432$

$\theta=75°52'$

Answer: (b)

116) Magnitude of unit vector=1

$1=[(0.5)^2+(0.8)^2+c^2]^{1/2}$

$c^2=0.11$

$c=\sqrt{(0.11)}$

Answer:(b)

117) $|\mathbf{A}+\mathbf{B}|^2=|\mathbf{A}-\mathbf{B}|^2$

$A^2+B^2+2AB\cos\theta=A^2+B^2-2AB\cos\theta$

$4AB\cos\theta=0$

$\therefore \theta=90°$ So angle between vector A and B is 90°

Answer: (d)

118) from given $(\mathbf{A}+\mathbf{B})^2=\mathbf{C}^2$

$A^2+B^2+2AB\cos\theta=C^2$

$3^2+4^2+2(3)(4)\cos\theta=5^2$

$\therefore 2AB\cos\theta=0$

$\cos\theta=0$thus angle between **A** and **B** is 90°

Answer:(a)

119) $|A\times B|=AB\sin\theta$

$A.B\ AB\cos\theta$

given $|A\times B|=AB\sqrt{3}$

$AB\sin\theta=\sqrt{3}(AB\cos\theta)$

$\tan\theta=\sqrt{3}$

$\therefore \theta=60°$

$\therefore |A+B|=(A^2+B^2+2AB\cos 60)1/2$

$|A+B|=(A^2+B^2+AB)\ 1/2$

Answer: (b)

120) $B\times A=C$

Direction of vector C is perpendicular to both **A** And **B**

Now $C.A=CB\cos 90$

Thus $(B\times A).A=0$

Answer: (d)

121) When a vector is multiplied by a scalar result is vector

Answer: (c)

122) power P=F S

$P=(6i-4j+3k).(20i+15j-5k)$

$P=45\ J/s$

Answer: (a)

123) given $A\times B=B\times A$

$A\times B-B\times A=0$

$Now\ -B\times A=A\times B$

$\therefore A\times B+A\times B=0$

$\Rightarrow A\times B=0$

$\Rightarrow \theta=0,\pi$ and $2\pi$

from given option θ=π

Answer: (c)

124) $v=u+at$

$v=3i+4j+(0.4i+0.3j)10\ v$
$=(3+4)i+(4+3)j$

$v=(7)i+(7)j$

$|v|=\sqrt{7^2+7^2}=7\sqrt{2}$

Answer: (a)

125) Given **P** and **Q** are perpendicular for perpendicular vector **P**.**Q**=0

$(ai+aj+3k)\cdot ai-2j-k)=0$

$a2-2a-3=0$

$a=3$

Answer: (d)

126) Since $A^2+B^2=C^2$

Therefore A and B are the sides of right angle triangle. Thus angle between A and B is π/2

Answer: (d)

127) Let **R**=**P**+**Q**

$|\vec{R}|=\sqrt{P^2+Q^2+2PQ\cos\theta}$

Given that |**R**|=|**P**| and |**P**|=|**Q**|

$P^2=P^2+P^2+2P^2\cos\theta$

$2\cos\theta=-1\rightarrow\cos=-\dfrac{1}{2}\rightarrow\theta=120°$

Answer: (a)

128) $A.B=AB\cos\theta$

Dot product= $(3)(5)\cos 60$

Dot product= $(3)(5)(1/2)=7.5$

Answer: (b)

129) If vectors are perpendicular dot product is zero, take dot product from the options, it is zero then that option is correct

Answer: (c)

130) $|A|^2=|B|^2+|B|^2+2|B||B|\cos\theta$

$25=16+9+2(4)(3)\cos\theta$

$25=25+24\cos\theta$

$\Rightarrow \cos\theta=0$

$\therefore \theta = \pi/2$
Answer:(c)

131) Alternative (d) is correct because the triangle is not formed if third side is greater than the sum of first two
Answer: (d)

132) cos of angle between them may be between -1 to +1 thus correct option is (d)
Answer:(d)

133) Since angle between 1st and nth vector is 2π vectors are symmetrically distributed hence resultant will be zero
Answer: (d)

134) Dot product of perpendicular vector is zero. Out of given option, option (d) is correct
Answer:(d)

135) Because if we add 2N and 2N it becomes four, so in 1st option four comes in this way.
If we subtract 2N from 6N then they are equal to 4
In third option there is 4N present in start so it is equal to 4.
The resultant of option d cannot be zero because if we add 2 and 8 it become 10, if we subtract 2 from 8 then it become 6..
**Note that if we divide 2N by 8N then it is equal to 4, but division of vector is not possible..**
Answer (d)

136) Torque is perpendicular to plane containing **r** and **F** thus dot product of rand **T** is zero and dot product of **F** and **T** is zero. option (d) is correct
Answer: (d)

137) Case I when forces are parallel let resultant force be at distance x from force P
p(x)=3P(40-x)
x=30 ( 30cm from P)
Now force P is reversed let the resultant force be at distance y from force P
-p(y)=3P(40-y)
y=60 cm resultant shift through distance of (60-30)=30cm
Answer: (a)

138) Let **A** be the smaller force and **B** be bigger force
Given that resultant force **R** is perpendicular to **A** thus vector **B** is hypotenuse of triangle
Thus $B^2=R^2 + A^2$

$$R^2 = B^2 - A^2$$
$$R^2 = (B + A)(B - A)$$
$$8^2 = (16)(B - A)$$
$$\frac{64}{16} = (B - A)$$
$$\therefore B - A = 4$$

only option (a) satisfy above condition
Answer: (a)

139) $A = B + C$

$$so\ B = A - C$$
$$B^2 = A^2 + C^2 - 2AC\cos\theta$$
$$4^2 = 5^2 - 2 \times 5 \times 3 \cos\theta$$
$$\cos\theta = 3/5$$

Answer:(a)

140) **B** is perpendicular to **A** if **A**.**B**=0
Apply this condition to all the four expressions for **B** in the given question, the expression (d) will satisfy the above condition
Answer: (d)

141) The vectors must form a right Δ in which **A**⊥**C** and |**A**|=|**C**|
∴ The other two angles must be 45° each
Answer: (a)

142) A+B=-B
A=-2B Thus Angle is zero Answer (d)

143) Let the resultant force at a distance of x from 4N force
Thus 4x=6(4-x)
∴ x=2.4 m
Answer: (a)

144) (b)

145) (a)

146) A × B=ABsinθ
Answer: (c)

147) (c)

148) |A × B|=Area of parallelogram
Answer: (a)

149) (a)

150) given **A** +2**B** is perpendicular to **A**

$$\therefore (\vec{A} + 2\vec{B}).\vec{A} = 0$$
$$A^2 + 2AB\cos\theta = 0$$

Also

$$(\vec{A} + \vec{B}).(\vec{A} + \vec{B}) = \vec{R}.\vec{R}$$
$$A^2 + B^2 + 2AB\cos\theta = R^2$$
$$\therefore B^2 = R^2 \Rightarrow B = R$$

Answer: (c)

151) Projection of vector **A** over **B** is component of vector A along the direction of vector B
Answer:(b)

152) (d)

153) (a)

154) Let P = A+B and Q = A-B Let θ be the angle between them

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$
$$3A^2 + B^2 = A^2 + B^2 2AB + A^2 + B^2 - 2AB + 2(A + B)(A - B)\cos\theta$$
$$3A^2 + B^2 = 2A^2 + 2B^2\ 2(A^2 - B^2)\cos\theta$$
$$A^2 - B^2 = 2(A^2 - B^2)\cos\theta$$
$$1 = 2\cos\theta$$
$$\theta = \pi/3$$

Answer: (b)

155) force must be 1000 dyne , no option is given
Answer:(d)

156) (b)

157) (c)

158) (c)

159) Unit vector along (**A**+**B**)=$\frac{A+B}{|A+B|}$

Answer: (a)